

Name: _____ ID #: _____

INSTRUCTIONS:

- Unless otherwise stated, your answers should be at most 1 or 2 sentences (excluding work.)
- This is a closed book, closed notes exam.
- Check to see that you have **12** pages including this cover and scratch pages.
- Read all the problems before starting work.
- Think before you write.
- If you leave a question blank or write just “I do not know,” you get 25% automatically.
- Good luck!!

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty.

Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that if I am caught cheating (either receiving or giving unauthorized aid) I will get a “Q” grade for this course, and a letter will be sent to the Committee on Academic Standing and Appeals (CASA) requesting that an academic dishonesty notation be placed on my transcript. Further action against me may also be taken.

Signature: _____

Problem	Score	Maximum
1		8
2		8
3		10
4		5
5		15
6		18
7		24
8		12
Total		100

Problem 1. (8 points)

Circle one of the choices below. Then justify using at most three sentences.

The number of *regular* languages over alphabet $\{a, b\}$ is

- (a) finite,
- (b) countably infinite,
- (c) uncountably infinite.

Justification: _____

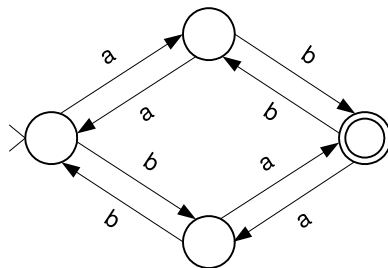
Problem 2. (8 points)

Describe in several words what the language generated by the following automata or grammar is.

(A)

$S \rightarrow aB \mid bA$
$A \rightarrow a \mid aS \mid BAA$
$B \rightarrow b \mid bS \mid ABB$

(B)



Problem 3. (10 points)

Suppose that I have a computer with 1 GB, which is 2^{30} bytes, or 2^{33} bits of memory. Note that the memory stores both the program and the program data. Suppose that the computer runs at 1GHz, i.e., 2^{30} operations every second. How long would it take us to solve the halting problem on this machine?

Problem 4. (5 points)

According to Arnold Rosenberg, what is the hardest natural language?

Problem 5. (15 points)

Circle **T** or **F** for the following. For each, provide a proof or counterexample. Please use at most 2-3 sentences per each.

T **F** If L is finite, then it must be that $|L^*| > |L|$.

Proof or counterexample: _____

T **F** Every subset of a regular language is regular.

Proof or counterexample: _____

T **F** There exists an undecidable language L such that L^* is regular.

Proof or counterexample: _____

Problem 6. (18 points)

Circle **T** or **F** for the following. No justification is necessary.

(1) All finite languages are decidable.

T **F**

(2) If there exists $xyz \in L$ such that $xy^n z \in L$ for all $n \geq 0$, then L is regular.

T **F**

(3) If L is a context-free language, then $\{w : w \in L \text{ and } |w| > 2001\}$ must be context-free.

T **F**

(4) If L_1 is regular and $L_1 \cap L_2$ is context-free, then L_2 must be context-free.

T **F**

(5) If L is countably infinite, then L^* is uncountably infinite.

T **F**

(6) If M is a DFA, then $L(M)$ is infinite if and only if M 's state-and-transition graph contains a cycle. (Self-loops count as cycles.)

T **F**

Problem 7. (24 points)

Complete the following tables with YES or NO as appropriate. No explanations needed. Recall that the complement of a co-r.e. language is an r.e. language.

Language:	regular	CF	decidable	r.e.	co-r.e.
$\{a^n b^{2n} a^{3n} \mid n \geq 0\}$					
$\{a^n b^{2n} \mid n \geq 0\}$					
$\{“w” \mid \text{some Turing machine } M \text{ halts on } w\}$					

Language:	decidable	r.e.	co-r.e.
$\{“M” “w” \mid \text{linearly bounded automata } M \text{ halts on input } “w”\}$			
$\{“M” \mid M \text{ is a TM that accepts } w \text{ whenever it accepts } w^R\}$			
$\{“M” “w” \mid M \text{ never moves its head left on input } “w”\}$			

Problem 8. (12 points)

Prove that the language $L_\infty = \{ \text{"}M\text{"} \mid L(M) \text{ is infinite} \}$ is not decidable by using a **reduction**, as in class.

Proof guidelines: Show that if L_∞ were decidable, then the halting problem would also be decidable.

(a) Draw and label a picture showing your reduction.

(b) Please describe the Turing Machine M' that you feed into the "black box."

(c) Write any additional text that might make the proof clearer. (You can leave blank if your first two parts say it all...)

