CSE 150 – Fundamentals of Computer Science: Honors **Final Exam**
Michael A. Bender **Tuesday, December 20, 2005**

Name: ______________________ ID #: ______________________

INSTRUCTIONS:

- Unless otherwise stated, your answers should be at most 1 or 2 sentences (excluding work.)
- This is a closed book, closed notes exam.
- Check to see that you have **15** pages (11 problems, 1 cover, and 3 scrap pages.)
- Read all the problems before starting work.
- Think before you write.
- If you leave a question blank or write just "I do not know," you get 25% automatically.
- Good luck!!

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty.

Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that if I am caught cheating (either receiving or giving unauthorized aid) I will get a "Q" grade for this course, and a letter will be sent to the Committee on Academic Standing and Appeals (CASA) requesting that an academic dishonesty notation be placed on my transcript. Further action against me may also be taken.

Signature: ______________________

Problem	Score	Maximum
1		8
2		5
3		5
4		10
5		5
6		16
7		8
8		12
9		12
10		10
11		9
Total		100

Problem 1. (8 points) Indicate whether the following statements are true or false (for all sets A and B). Circle **T** (True) or **F** (False). No justification is required.

1. **T** **F** $\emptyset \in \emptyset$.

2. **T** **F** $\emptyset \subseteq \emptyset$.

3. **T** **F** $\emptyset \in \{\emptyset\}$.

4. **T** **F** $\emptyset \subseteq \{\emptyset\}$.

Problem 2. (5 points) Let R be a binary relation. Let $T(R)$ be the transitive closure of R. Let $S(R)$ be the symmetric closure of R. Circle **T** (True) or **F** (False) and justify your answer with a proof or counterexample. No credit will be given for answers without justification.

1. **T** **F** $T(S(R)) \subseteq S(T(R))$.

Problem 3. (5 points) For each of the following problems, find the smallest two finite sets A and B such that:

1. $(A \bigcap B) \in A$, $B \subset A$, and $P(B) \subseteq A$.

Problem 4. (10 points) Group the following propositional formulas into the smallest number of groups such that all propositions in a single group are logically equivalent.

- Proposition A $\neg\neg(P \bigwedge Q)$.
- Proposition B $P \Rightarrow \neg\neg Q$.
- Proposition C $(P \bigwedge Q \bigwedge R) \bigvee (\neg R \bigwedge Q \bigwedge P)$.
- Proposition D $\neg(\neg P \bigvee \neg Q)$.
- Proposition E $\neg Q \Rightarrow \neg P$.

Problem 5. (5 points) Determine whether the following formulas are true or false when interpreted over the set of integers ($\mathbb{Z}$). Circle **T** (True) or **F** (False). Justify in at most two sentences. No credit will be given without justification.

1. **T** **F** $\forall m \exists n \, ((m + n = 4) \bigwedge (m - n = 1))$.

Problem 6. (16 points) Set S contains 16 distinct 8-bit binary numbers.

1. How many subsets of S exist?____________________
   (You should give the answer in exponential form.)

2. The largest sum of any of the subsets of S could require more than 8 bits to represent.

   What is the maximum number n of bits required to represent this largest sum?__________

   How many different values can an n-bit number have?____________________
   (You should give the answer in exponential form.)

3. Prove that there exist two distinct subsets of S whose sums are equal.

   General Proof Technique____________________

4. Prove that there exist at least 16 (Hint: $16 = 2^4$) distinct subsets of S whose sums are equal.

   General Proof Technique____________________

Problem 7. (8 points) Let $F(x, y)$ mean "x can fool y."

Express the following statements using predicate logic in terms of $F(x, y)$.

1. Everybody can fool Bob.__________________________

2. Nancy can fool no one but Fred.__________________________
   (Nancy **can** fool Fred.)

3. Everybody can fool somebody.__________________________

4. **T F** Are the three statements above logically consistent when taken together? Circle **T** (True) or **F** (False) and justify your answer briefly. No credit will be given for answers (to Part 5) without justification.

Problem 8. (12 points) In the show "Smallville", Clark Kent faces seemingly improbable events in every episode. Here are some examples:

- A: An ancient artifact turns out to be a piece of alien technology. Probability $= \frac{1}{7}$
- B: Radiation from kryptonite grants someone superhuman powers. Probability $= \frac{1}{8}$
- C: Radiation from kryptonite causes someone to become sick or die. Probability $= \frac{1}{5}$
- D: Someone dies and no one cares. Probability $= \frac{4}{5}$
- E: Clark's parents get a new car (to replace the one that exploded in the last episode). Probability $= \frac{9}{10}$

For 1-2, give the probabilities of the events. Assume that events A, B, C, D, and E are mutually independent.

1. $P(C \bigcap D \bigcap E)$

2. $P(E|B)$

For 3-4, give the probabilities of the events or upper and lower bounds when that's all that you can conclude. Assume that events A, B, C, D, and E are **pairwise** independent.

3. $P(B \bigcap C \bigcap E)$

4. $P(A|B)$

Problem 9. (12 points) Recall the Towers of Hanoi. The goal is to move all discs from one peg to another with the following rules:

- The discs all begin on the first of three pegs, sorted in decreasing size as you go up (i.e., with the smallest on top).
- No two discs are of the same size.
- A disc may not be placed on top of a smaller disc.
- The only "move" allowed is to take the top disc from one peg and place it onto a different peg. This counts as a **single** move.

Let $T(n)$ be the number of moves required to move all the discs onto a different peg.

1. What is the full recurrence for $T(n)$?
   Note: Recall that a recurrence includes the base cases.

2. What is the solution for Part 1?

Suppose that we add **another** legal move. You now may also move the top **two** discs from one peg and place them onto a different peg. This also counts as a **single** move.

3. Now what is the full recurrence for $T(n)$?

4. What is the solution for Part 3?

Problem 10. (10 points) Answer the following counting questions:

1. How many ways can you arrange $\boldsymbol{s}$ **stars** and $\boldsymbol{b}$ **bars** in a single line?

   Answer in "choose" notation______________________________

   Answer in "factorial" notation______________________________

2. How many ways can you arrange $\boldsymbol{s}$ **stars**, $\boldsymbol{b}$ **bars**, and $\boldsymbol{c}$ **cars** in a single line?

   Answer in "factorial" notation______________________________

Problem 11. (9 points) A, B, and C are events.

$P(A) = 1/2$.
$P(B) = 3/4$.
$P(C) = 2/3$.
$P(A \bigcap B) = 1/3$.
$P(A \bigcap C) = 3/7$.
$P(B \bigcap C) = 1/2$.
$P(A \bigcap B \bigcap C) = 1/4$.

Find the following probabilities:

1. $P(C|A)$

2. $P(A|C)$

3. $P(C|B \bigcap A)$

Scratch Paper

Scratch Paper

Scratch Paper