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CSE 150, Fall 2006

Assignment #1
Due Tuesday, September 26, 2006 **at beginning of class**

Each of the problems should be solved on a separate sheet of paper to facilitate grading. Please don't wait until the last minute to look at the problems.

I encourage you to try to write up solutions using latex. To get latex running on windows, install Miktex and WinEdt on your PC. For drawing figures, I recommend using the unix tool xfig.

Problem 1 Set Review
For each of the following statements about sets determine whether it is always true (also provide an example), or only sometimes true (also provide an example and counterexample). Please provide an explanation.

1. $A \in P(A)$
2. $A \subseteq P(A)$
3. $(\,|A| \leq |B|\,) \Rightarrow (A \subseteq B\,)$
4. $(A \subseteq B\,) \Rightarrow (\,|A| \leq |B|\,)$

Problem 2
Find the smallest two finite sets A and B for each of the four conditions.
Note: The smallest sets may not be unique.

1. $A \in B$, $A \subseteq B$, and $P(A) \subseteq B$.
2. $(\mathbb{N} \bigcap A) \in A$, $B \subset A$, and $P(B) \subseteq A$.
3. $A \subseteq (P(P(B)) - P(A))$.
4. $A \supseteq (P(P(B)) - P(A))$.

Problem 3
Prove or disprove (by providing a counterexample) each of the following properties of binary relations:
Let $S(A)$ be the symmetric closure of set A. Let $T(A)$ be the transitive closure of set A.
For every binary relation R,

1. $T(S(R)) \subseteq S(T(R))$
2. $S(T(R)) \subseteq T(S(R))$

Problem 4
Show that each function $f : \mathbb{N} \rightarrow \mathbb{N}$ has the listed properties.

1. $f(x) = 2x$ (one-to-one but not onto)
2. $f(x) = x + 1$ (one-to-one but not onto)
3. $f(x) =$ if x is odd then $x - 1$ else $x + 1$ (bijective)

Problem 5

Show that the product $(a+bi)(c+di)$ of two complex numbers can be evaluated using just three real-number multiplications. You may use a few extra additions.

Problem 6

Given a function $f : A \to A$. An element $a \in A$ is called a *fixed point* of f if $f(a) = a$. Find the set of fixed points for each of the following functions.

Note: for help on modular arithmetic, please speak on Roman on Thursday.

1. $f : A \to A$ where $f(x) = x$.
2. $f : \mathbb{N} \to \mathbb{N}$ where $f(x) = x + 1$.
3. $f : \mathbb{N}_6 \to \mathbb{N}_6$ where $f(x) = 2x \mod 6$.
4. $f : \mathbb{N}_6 \to \mathbb{N}_6$ where $f(x) = 3x \mod 6$.

Problem 7

Let $f(x) = x^2$ and $g(x, y) = x+y$. Find compositions that use the functions f and g for each of the following expressions.

1. $(x + y)^2$
2. $x^2 + y^2$
3. $(x + y + z)^2$
4. $x^2 + y^2 + z^2$