CSE 150 – Fundamentals of Computer Science: Honors **Midterm Exam**
Michael A. Bender **October 20, 2005**

Name: _____ ID #: _____

INSTRUCTIONS:

- Unless otherwise stated, your answers should be at most 1 or 2 sentences (excluding work.)
- This is a closed book, closed notes exam.
- Check to see that you have **12** pages including this cover and scratch pages.
- Read all the problems before starting work.
- Think before you write.
- If you leave a question blank or write just "I do not know," you get 25% automatically.
- Good luck!!

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty.

Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that if I am caught cheating (either receiving or giving unauthorized aid) I will get a "Q" grade for this course, and a letter will be sent to the Committee on Academic Standing and Appeals (CASA) requesting that an academic dishonesty notation be placed on my transcript. Further action against me may also be taken.

Signature: _____

Problem	Score	Maximum
1		10
2		10
3		15
4		10
5		10
6		15
7		15
8		15
Total		100

Problem 1. (10 points) Let $U = \{a, b, c, d, e\}$. Let $R = \{(a, b), (c, b), (c, d), (d, c), (e, a)\}$ be a binary relation on U. Let $T(R)$ be the transitive closure of R. Let $S(R)$ be the symmetric closure of R. Provide a graphical representation of:

1. R.

2. $T(R)$.

3. $S(T(R))$.

Problem 2. (10 points) For each of the following problems, find the smallest two finite sets A and B such that:

1. $A \supseteq (P(P(B)) \bigcup A)$.

2. $A \supseteq (P(P(B)) \bigcap A)$.

Problem 3. (10 points) Indicate whether the following statements are true or false (for all sets A and B). Circle **T** (True) or **F** (False). No justification is required.

1. **T** **F** $\emptyset \in \emptyset$.

2. **T** **F** $\emptyset \subseteq \emptyset$.

3. **T** **F** $|P(A \times B)| = |P(A) \times P(B)|$.

"Not all things worth counting are countable and not all things that count are worth counting."
– Albert Einstein

"Only two things are infinite, the universe and human stupidity, and I'm not sure about the former."
– Albert Einstein

Problem 4. (10 points)

For each of the following statements about sets, circle **T** (True) or **F** (False). No justification required.

1. **T** **F** $(A \bigcup B)$ is uncountable $\Rightarrow$ A and B are both uncountable.

2. **T** **F** $P(A)$ is countable $\Rightarrow$ A is countable.

Problem 5. (10 points) Determine whether the following formulas are true or false when interpreted over the set of integers ($\mathbb{Z}$). Circle **T** (True) or **F** (False). No justification necessary.

1. **T** **F** $\exists n \forall m \, (n < m^2)$.

2. **T** **F** $\forall m \forall n \exists p \; (m + n = 2p)$.

3. **T** **F** $\forall m \forall n \exists p \; \neg(m + n = 2p)$.

Problem 6. (15 points) Suppose n people go to a party. Some people know each other. Everyone knows themselves. Assume knowing is symmetric (if Bob knows George, then George knows Bob.)

1. Suppose that everyone knows at least one other person (in addition to themselves.) Prove that at least 2 people know the same number of people.

2. Prove that at least 2 people know the same number of people without the restriction in Part 1. (Everyone still knows themselves).

Problem 7. (15 points) Let $|S| = n$. Let $P(S)$ be the powerset of set S. Namely that $P(S)$ is the set of all subsets of S. In class you learned $|P(S)|$ in terms of n.

1. State $|P(S)|$ in terms of n.

2. Prove Part 1 by induction on n. You will be graded on the correctness of the proof as well as the quality of it. Make certain all information that should be in an induction proof is provided. Write clearly.

Problem 8. (15 points) Consider the following quotes. Answer the questions below each quote and provide only the general idea for a proof.

1. "*Hamlet:* (takes the skull)

   Alas, poor Yorick! I knew him, Horatio: a fellow of infinite jest, of most excellent fancy."

   – William Shakespeare's *Hamlet*

   Consider the (infinite) set of all jests (jokes) that can be told by people. Is this set countable?
   Note: A jest can only be so long.[1]

2. "*Charmain:* "Is this the man? Is't you, sir, that know things?"

   Soothsayer: "In nature's infinite book of secrecy/A little I can read."

   – William Shakespeare's *Antony and Cleopatra*

   Consider the set of all infinite books (all books of infinite length). Is this set countable?

[1] It has a finite gestation period.

Scratch Paper

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