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CSE 350 – Theory of Computation: Honors, Spring 2007

Problem Set #2

Due Monday, March 5th, 2007

Please don't wait until the last minute to look at the problems. Please cite any collaborators and any sources. The homework is to be submitted at the beginning of the class. Only latex submissions will be accepted. The material in this problem set is covered in Chapter 1 (Section 1.3,1.4) of Sipser.

Problem 1

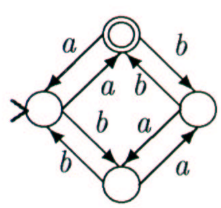
Describe informally the language represented by the following regular expressions and construct the corresponding NFA.

(A) $((a \cup b \cup e)(a \cup b \cup e)b)^*(a \cup b \cup e)(a \cup b \cup e)$

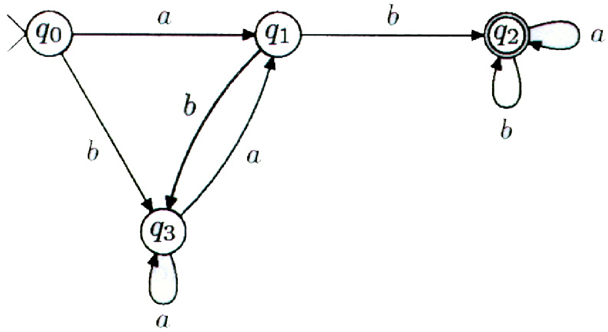
(B) $b\emptyset^* a \cup ab \cup ba\emptyset ba$

Problem 2

Give the regular expression and language represented by the following DFA.



Problem 3



Using the construction described in class, convert the NDFA pictured above into a regular expression. Be sure to show each step.

Problem 4

Write regular expressions for each of the following languages over the alphabet {0,1}

(a) The set of all strings in which every pair of adjacent 0's appear before any pair of adjacent 1's.

(b) The set of all strings with an equal number of 0's and 1's such that no prefix has two more 0's than 1's nor two more 1's than 0's.

(c) The set of all strings not containing 101 as the substring.

Problem 5

For the following languages, give a DFA if possible. If it is not possible, then prove that it is not possible.

The input alphabet is binary. Let a be the number of 1's and b the number of 0's.

(a) $|a - b|$ is even

(b) $a > b$

Problem 6

(A) True of false: every subset of a regular language is regular. Justify.

(B) True of false: the union of two co-finite languages is regular. Justify.

Problem 7

In the following, I summarize a comment from the book. Just because a language appears to require unbounded memory doesn't mean it is necessarily so. Consider two languages over the alphabet $\Sigma = \{0, 1\}$:

(1) $C = \{w|w$ has an equal number of 0s and 1s$\}$, and

(2) $D = \{w|w$ has an equal number of occurrences of the substrings 01 and 10$\}$.

Thus, $010 \in D$ but $1010 \notin D$.

Prove that C is not regular, but that D is.

Problem 8

Use the pumping lemma to prove that the following languages are not regular:

(a) $A_1 = \{a^i b^j c^k;$ for $i, j, k \geqslant 0$ and either $i = j$ or $j = k\}$.

(b) $B_1 = \{\omega$ such that ω has exactly twice as many 1's as 0's$\}$.