Michael A. Bender

CSE 350 – Theory of Computation: Honors, Spring 2007

Problem Set #3
Due Wednesday, March 14, 2007

Please don't wait until the last minute to look at the problems. Please cite any collaborators and any sources. The homework is due at the beginning of the class. The material in this problem set is covered in Chapter 2 of Sipser.

Problem 1

Give a context-free grammar that generates the language.

$$A = \{a^i b^j c^k | i, j, k \geq 0 \text{ and either } i = j \text{ or } j = k\}.$$

Is your grammar ambiguous? Why or why not?

Problem 2

Use the canonical method described in the class and in the textbook to construct a PDA accepting the language generated by the following grammar: $G = (V, \Sigma, S, P)$ where $V = \{S, T, F\}, \Sigma = \{a, +, *, (,)\}$, and

$$P = \{S \rightarrow S + T | T, T \rightarrow T * F | F, F \rightarrow (S) | a\}.$$

Problem 3

Show that the following languages are context-free.
(A) $\{a^m b^n \ : m \neq n\}$.
(B) $\{a, b\}^* - \{a^n b^n \ : n \geq 0\}$.
(C) $\{w \in \{a, b\}^* \ : w = w^R\}$.

Problem 4

Please prove using the pumping lemma that $a^n b^n c^n$ is not context free. (You can try to figure this out yourself or look in the book, whichever will help you the most.)

Problem 5

Please prove that (a) context free languages are closed under union, and that (b) context free languages are *not* closed under intersection. (Again, you can try to figure this out by yourself or look in the book, as you choose.)
Hint: For (b), try to a counterexample, showing two context free languages whose intersection isn't context free.

Problem 6

Show that the following languages are not context-free.

(A) $\{a^p : p \text{ is a prime }\}$.

(B) $\{a^{n^2} : n \geq 0\}$.

(C) $\{www : w \in \{a,b\}^*\}$.

(D) $\{w \in \{a,b,c\}^* : w \text{ has equal numbers of } a\text{'s, } b\text{'s, and } c\text{'s }\}$.

(E) $\{ww : w \in \{a,b\}^*\}$.

Problem 7

Determine whether each of the following languages is context-free or not. Support each of your answers with a proof.

(A) $\{a^n b^n c^n d^n : n \geq 0\}$

(B) $\{a^i b^j c^k : i = j \text{ or } i = k \text{ or } j = k\}$

To do next problem set... You don't have to worry about until after the midterm.

Problem 5

Clean up the following grammar by removing useless, epsilon and unit productions and then convert it to Chomsky Normal Form.

$S \leftarrow AB|C$

$A \leftarrow 0B|C|\epsilon$

$B \leftarrow 1|A0|\epsilon$

$C \leftarrow AC|C1$

$D \leftarrow AD|0$

Problem 6

Using the CYK algorithm as discussed in class find weather the string abaaa is present in the following grammar

$S \leftarrow AS|b$

$A \leftarrow AA|a|b$

$B \leftarrow BB|a$

Problem 7

Show that, if G is a CFG in CNF, then for any string $w \in L(G)$ of length $n \geq 1$, exactly 2n-1 steps are required for any derivation of w.