

**CSE 350 – Theory of Computation: Honors, Spring 2007**

Problem Set #4

Due Wednesday, April 25, 2007

Please don't wait until the last minute to look at the problems. Please cite any collaborators and any sources. The homework is due at the beginning of the class. The material in this problem set is covered in Chapter 3 of Sipser.

Problem 1

A queue machine is like a PDA except that it has a queue instead of a stack. The machine accepts by emptying its queue and entering a final state (as we designed for a PDA). Give a rigorous formal definition of a queue machine (analogous to the 6-tuple definition of a PDA in the textbook or similar definition from class). Prove that queue machines and Turing machines are equivalent in power.

Problem 2

We have usually limited our discussions of languages to answering whether a string is in a language or not. Let's compute some functions.

Design a Turing machine that, given two numbers represented in binary, computes the sum of the numbers. For simplicity, you may assume you have a two-tape Turing machine. The two numbers are encoded on the first tape and the result must be on the second tape. When your TM halts, the pointer of the second tape must be pointing at the beginning of the result.

In designing the TM, you should give the alphabet, the transitions, and all components of the formal definition. You should also explain in words how it works, and you should give the running time as a function of the input size.

Problem 3

Show that the collection of Turing-recognizable languages is closed under the operation of

- (a) Union
- (b) Concatenation
- (c) Star
- (d) Intersection

Problem 4

Compare the running time (i.e., number of steps) of the following equivalent machine

- (A) Single tape TM and  $k$ -tape TM.
- (B) two-stack machine and single tape TM
- (C) two-stack machine and a four-counter machine.