CSE 350 – Theory of Computation: Honors **Midterm Exam**
Michael A. Bender **Tuesday, March 21, 2006**

Name: _____ ID #: _____

INSTRUCTIONS:

- Unless otherwise stated, your answers should be at most 1 or 2 sentences (excluding work.)
- This is a closed book, closed notes exam.
- Check to see that you have **10** pages including this cover and scratch pages.
- Read all the problems before starting work.
- Think before you write.
- If you leave a question blank or write just "I do not know," you get 25% automatically.
- Good luck!!

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty.

Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that if I am caught cheating (either receiving or giving unauthorized aid) I will get a "Q" grade for this course, and a letter will be sent to the Committee on Academic Standing and Appeals (CASA) requesting that an academic dishonesty notation be placed on my transcript. Further action against me may also be taken.

Signature: _____

Problem	Score	Maximum
1		16
2		28
3		10
4		20
5		14
6		12
Total		100

Problem 1. (**16** points)
Let language $L_1 = \{w \in \{a, b\}^* \,|\, w \text{ has } bab \text{ as a substring}\}$.

(a) Draw a deterministic finite automaton (DFA) accepting L_1.

(b) Give a regular expression accepting L_1.

Problem 2. (**28** points)
Are the following languages **regular**, **context free**, or **neither**? Please give a one-sentence justification.

R **CF** **N** $L_1 = \{a^n b^n | \, n \leq 350\}$

R **CF** **N** $L_2 = \{a^n b^n | \, n \geq 350\}$

R **CF** **N** $L_3 = \left\{xyzy^R x | \, x, y, z \in \{a, b\}^*\right\}$

R **CF** **N** $L_4 = \left\{a^i b^j c^k | \, i = j \text{ or } j = k \text{ or } i = k\right\}$

R **CF** **N** $L_5 = \left\{a^i b^j c^k | \, i \leq j \leq k\right\}$

R **CF** **N** $L_6 = \{w \in \{a, b\}^* \, | \, w \text{ has an equal number of } a\text{'s and } b\text{'s }\}$

R **CF** **N** $L_7 = \{w \in \{a, b\}^* \, | \, w \text{ has an equal number of } ab\text{'s and } ba\text{'s }\}$

Problem 3. (**10** points)
Circle one of the choices below. Then justify using at most three sentences.

The number of regular languages over alphabet $\{a, b\}$ is

(a) finite,

(b) countably infinite,

(c) uncountably infinite.

Justification: _____

Problem 4. (**20** points)
True or **false**. No explanation needed.

T **F** The intersection of two context-free languages is a context-free language.

T **F** The intersection of two regular languages is a regular language.

T **F** The complement of a regular language is a regular language.

T **F** Deterministic finite automata (DFA) have the same computational power as nondeterministic finite automata (NDFA).

T **F** Deterministic push-down automata have the same computational power as nondeterministic push-down automata.

Problem 5. (**14** points)
Please answer the following questions with proofs.

- Let $L_2 = \{w | w$ is a regular expression over a and $b\}$.

  Thus, the alphabet is $\Sigma = \{$‘a’,‘b’,‘ε’,‘$\emptyset$’,‘$\cup$’,‘(’,‘)’$\}$.
  Recall that R is a regular expression if R is
  (1) a or b, (2) ε, (3) $\emptyset$, (4) (R_1), (5) $(R_1)^*$, (6) $(R_1 \cup R_2)$, (7) $(R_1 R_2)$,
  where R_1 and R_2 are regular expressions.

  Is L_2 regular? _____
  Explain why or why not?

  Is L_2 context free? _____
  Explain why or why not?

Problem 6. (**12** points)
Please answer the following questions with proofs.

- Let $L_3 = \{w | w \text{ is a context-free grammar}\}$.

  Thus, the alphabet is $\Sigma = \{$ 'A', ...,'Z', 'a', ..., 'z', '$\rightarrow$', '&' $\}$.

  Recall that a context-free grammar is zero or more productions (which we separate here using '&'). Each production is a single capital letter, followed by '$\rightarrow$', followed by either 'ε' or a (nonempty) string of capital and/or lowercase letters.

  Example: Thus, the language $\{a, b\}^*$ is represented as string

  $$S \rightarrow \varepsilon \ \& \ S \rightarrow aS \ \& \ S \rightarrow bS$$

  Is L_3 regular? _____
  Explain why or why not?

  Is L_3 context free? _____
  Explain why or why not?

Scratch Paper

Scratch Paper

Scratch Paper