

CSE 505: Computing with Logic, Fall 2008

Homework 2

Due: **Mon., Oct 27**

[Please submit your answers either electronically via Blackboard, or leave a hardcopy in my mailbox.]

1. Let $F = \forall X \exists Y p(X, Y)$ and $G = \exists Y \forall X p(X, Y)$. State, for each of the following four formulas, whether it has a model with domain $\{a, b, c\}$; if not, briefly explain why not.

- (a) $F \wedge G$
- (b) $\neg F \wedge G$
- (c) $F \wedge \neg G$
- (d) $\neg F \wedge \neg G$

2. Consider the Herbrand universe consisting of the constants a, b, c , and d . Let \mathcal{I} be the Herbrand interpretation

$$\{p(a), p(b), q(a), q(b), q(c), r(b), r(c), r(d)\}$$

For which of the following formulas is \mathcal{I} a model? Justify your answer.

- (a) $\forall X q(X)$
- (b) $\forall X q(X) \vee r(X)$
- (c) $\exists X (p(X) \wedge q(X))$
- (d) $\forall \neg p(X) \vee q(X)$
- (e) $\forall X (q(X) \Rightarrow p(X))$
- (f) $\forall X (p(X) \Rightarrow q(X))$

3. Consider the following two definitions of the reachability relation as logic programs:

- a.** $reach(X, Y) \leftarrow edge(X, Y)$
 $reach(X, Y) \leftarrow edge(X, Z), reach(Z, Y)$
- b.** $reach(X, Y) \leftarrow edge(X, Y)$
 $reach(X, Y) \leftarrow reach(X, Z), reach(Z, Y)$

Consider the edge relation defined as:

$$\begin{aligned} &edge(a_1, a_2) \\ &edge(a_2, a_3) \\ &edge(a_3, a_4) \\ &edge(a_4, a_5) \end{aligned}$$

(continued on next page)

- (a) Using the T_P operator, find the least Herbrand model for the *reach* relation defined in **(a)**. Show each step of the iteration sequence used to compute the least model.
- (b) Using the T_P operator, find the least Herbrand model for the *reach* relation defined in **(b)**. Show each step of the iteration sequence used to compute the least model.
- (c) Consider a graph with vertices $\{a_1, a_2, \dots, a_n\}$ such that there is an edge between a_i and a_{i+1} for all $i \in \{1, 2, \dots, n-1\}$.
 - i. How many steps will it take to compute the least Herbrand model of the *reach* relation defined as **(a)**?
 - ii. How many steps will it take to compute the least Herbrand model of the *reach* relation defined as **(b)**?

For simplicity, you may assume that $n = 2^k$ for some $k \geq 1$.

4. Let P be the propositional formula

$$(p \vee q \vee \neg r) \wedge (r \vee q \vee \neg s) \wedge (\neg p \vee \neg s)$$

Let $G = (q \vee \neg s)$. Show that G is true in every model of P using refutation.