CSE 213 Fall 2007
Homework 2
Solutions

1)

i) Basis: $0 \in S_1$,
Induc: If $x \in S_1$, and x ends with 0; then $x1 \in S_1$,
If $x \in S_1$, and x ends with 1; then $x0 \in S_1$

ii) Basis: $1 \in S_2$
Induc: If $x \in S_2$, then $xS_1 \in S_2$

2)

i) (0,0), (2,0), (0,2), (1,1), (3,1), (1,3), (2,2), (4,0), (0,4), (3,3)
ii) T={(a,b)| a,b ∈ N and a+b is an even number}

3)

i) Basis: $<1,1> \in L_N$
Induc: If $L \in L_N$ then cons(head(L)+1, cons(head(L)+1,L))

4)

i) Basis: $(0,0) \in S$
Induc: If $(a,a) \in S$, then $(a+1,a+1) \in S$

5)

f(w,y) = isPrefix(reverse(w),reverse(y))

reverse(x)=
if x is λ then return λ
else if x=ay then return reverse(y)a
else if x=by then return reverse(y)b

isPrefix(x,y)=
if y is λ then return Yes
else if y=as and x=at or y=bs and x=bt then return isPrefix(t,s)
else return No

6)

i) Recall that for S_2, the empty string λ is the base case; using the inductive definition the next term to be added to S_2 take x and y as λ which will generate () as the next member of S_2 based on the rule (x)y. If we generate all subsequent using y=λ we'll add strings of the form (()) , ((())) , (((()))),….all these previous strings are profile balanced of the form x=(v)y for y=λ
Now we can make y take the value of all the strings already generated (i.e. Those that are already proven to be profiled based); we will have strings of the form: ()(), ()(()), () ((())) , …, (())(), (())(()), ….; therefore for y≠ λ for those y=(x) then we will have that x=(v)y is also profile balanced since is the concatenation of two profile base strings

ii) We have to show that every string w in S1 is profile base. This can be proven by induction on the number of application of the rules needed to generate w.
Induction Basis: The string generated without the application of the rules is the empty string, which by definition is profile balanced.
Suppose n>0 for the case (x) this rule can only generate profile balanced string regardless of the number of application of the rules; in the case of xy, this rule concatenates two strings already in the set; assuming the x and y are profile balanced, the concatenation of both of them generates a profile balanced string.
Initially we saw that any string generated by the rule (x) is profile balanced so any sting x, y in the set will be profile balance will be used to apply the xy rule.

7)

i) $S \rightarrow \wedge \mid aSa$

ii) $S \rightarrow aA \mid bB$
$A \rightarrow aB \mid bA \mid a$
$B \rightarrow bB \mid aA \mid b$

iii) $S \rightarrow aab \mid aaSb$

iv) $S \rightarrow \wedge \mid aSb \mid bSa \mid SS$