

Homework 1

Due in class on Friday, Sept 28, at the beginning.

(1) Informally explain that $A - B = A \cap B^c$, where B^c is a complement of B with respect to some universal set U .

(2) For sets A , B , and C , prove that $A - (B \cap C) = (A - B) \cup (A - C)$.

Note that a partition of a set S has subsets of S as members. These members/subsets follow some restrictions. A member of a partition of S is actually a subset of S . Let P_1 denote a partition of set $S = \{1, 2, 3, 4\}$ such that it has 3 members. For example $P_1 = \{ \{1, 2\}, \{3\}, \{4\} \}$ could be one such partition.

(3) Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A_1 = \{1, 2, 3, 4\}$, $A_2 = \{5, 6, 7\}$, $A_3 = \{4, 5, 7, 9\}$, $A_4 = \{4, 8, 10\}$, $A_5 = \{8, 9, 10\}$, $A_6 = \{1, 2, 3, 6, 8, 10\}$. Which of the following are partitions of A ?
 (i) $\{A_1, A_2, A_5\}$ (ii) $\{A_1, A_3, A_5\}$ (iii) $\{A_2, A_3, A_4\}$ (iv) $\{A_3, A_6\}$

(4) Let $S = \{a, b, c, d\}$. (i) What partition of S has the fewest members? (ii) The most members? (iii) List all partitions of S with exactly 2 members. Name them P_1, P_2, P_3, \dots etc.

(5) Let $A = \{1, 2, 3, \dots, k\}$ where $k > 0$. How many subsets of A are there such that they all have element 1 but not 2? Express your answer in terms of k . Give an informal proof supporting your answer.

(6) Let a, b, c, \dots , be k logical variables or propositions that assume values true or false (T or F). Consider k -variable boolean functions that map to a set $\{T, F\}$. How many different k -variable boolean functions that can be defined? Explain your answer. (Hint: Think about a truth table for k variables.)

(7) Consider a sequence $(0, -3, 6, -9, 12, -15, \dots)$. Give a function that creates this sequence.

(8) (i) List all strings over $X = \{a, b\}$ that are of length 3. (ii) Write all substrings of the string 'abaa'. (iii) Write all prefix and suffix strings for string 'ababba'.

(9) Let $X = \{a, b\}$, and let X^* denote set of all string over X . Give an inductive definition for a set of strings over X such that each string begins with bb .

(10) Give an inductive definition of a set of all integers from Z , such that they are divisible by 14 or 23 or both. (For a, b in Z , we say a is divisible by b if a/b is in Z . Here b is not zero.)