CSE 213 Fall 2007

Homework 2

Quiz-1: Friday, Oct 5, last 20 minutes. See announcement page for topics.

Exam-1: Friday Oct 12, whole class hour.

HW2 is due in class on Monday, Oct 8, at the beginning.

(1) Let X = {0, 1}, and let X^* denote set of all string over X.
(i) Give an inductive definition for a set S_1 of strings over X such that each string begins a 0, and is an alternating sequence of 0's and 1's. You may use conditions while writing recursive or inductive rules. Here S_1 = {0, 01, 010, 0101, ... }

(ii) Give inductive definition for another set S_2 such that each string in S_2 begins with 1, and is an alternating sequence of 0's and 1's. You must make use of S_1 while defining S_2.

(2) A set T of ordered pairs (a, b), where a, b are integers ≥ 0, is defined recursively as follows.

Rule-1: (0, 0), and (1, 1) are in T
Rule-2: If (a, b) is in T, then so is (a+2, b).
Rule-3: If (a, b) is in T then so is (a, b+2).

(i) List the first 10 elements/pairs (a, b) of set T in increasing order. Use a+b as value of a pair (a, b) for ordering purpose. Break ties arbitrary. Start with (0, 0).

(ii) The set T can be defined non-inductively using several other ways. Give one such definition that does not use induction or recursion but uses set comprehension. In other words,
T = { (a, b) | some property P is satisfied by a, b}. State property P.

(3) Consider the set of positive integers N^+ = {1, 2, 3, ...}. Let L_N denote a set of integer lists. Members of L_N are as follows.
< 1, 1 >, < 2, 2, 1, 1 >, < 3, 3, 2, 2, 1, 1>, < 4, 4, 3, 3, 2, 2, 1, 1> ... and so on.
Give inductive definition of set L_N. You may use cons, head, tail etc operations on a list.

Recursive functions: We studied this in CSE 113, but did not spend that much time on them in 213. Recursion and inductive definitions are closely related. These exercises are included for your practice. You may see a question either on a quiz, midterm exam or final. These are similar to exercises on pages 168-169. Their solutions are on the webpage/or in the book.

(4) Give an inductive definition for a set of ordered pairs (a, b), such that a = b. Here both a, b are positive integers.

(5) Let X = {a, b}. Give a recursive definition for function f(w, y) which tests if y is a suffix of w and returns Yes/No. w and y are strings over X^*. (Hint: See solution to Q5.c on page 169.)

Balanced Parenthesis: We studied 3 different definitions for sets of balanced parenthesis. These definitions defined sets S_1, S_2, and S_3. Here S_3 is a set of strings over { (,) } that are profile balanced.

(6) In class, (see notes) we studied a proof sketch that showed $S_2 = S_3$. The second part that proof showed $S_3 \subseteq S_2$. The proof goes something like this....
Induction basis and hypothesis as per notes.
Let w be a profile balanced string in S_3 such that |w| = n. Write w as xy, where x is the shortest nonempty profile balanced prefix of w.
(i) There are two cases to consider; y is not empty or y is empty. In both cases, we prove that we can write x = (v). Why do we need to do that ?

(ii) Notes do not include the proof of $S_1 = S_3$. Just write the first half of this proof which shows $S_1 \subseteq S_3$.

(7) **Grammars:** Consider languages defined over {a, b}. Give grammars for:

(i) L_1 = Set of strings with with even number of a's and no b's. Include null string.

(ii) L_2 = Set of strings with with even number of a's and 0 or more b's. Here b's could be anywhere. For example, bbabaaabbb is member of the set. (Hint: You will need one more nonterminal B, that derives strings with only b's.) Do not include null string.

(iii) $L_3 = \{a^{2n}b^n \mid n > 0\}$

(iv) L_4 = Set of strings with equal numbers a's and b's.