CSE 213 Fall 2007

Homework 3

HW3 is due in class on Monday, Oct 29, at the beginning.
Quiz-2: Friday, Nov 9, last 20 minutes. See announcement page for topics.
Exam-2: Monday Nov 19, whole class hour.

(1) Consider the set A = {1, 2, 3, 4, 5, 6} and relations R_1 and R_2 on A. List relations as sets of ordered pairs, find their domain and range (similar to function) and draw their graphs. Domain of a relation on A is a subset of A defined by Domain(R) = { x | (x, y) $\in$ R}.
Range(R) is a set { y | (x, y) $\in$ R}.
(i) R_1 = {(x, y) | (y is a multiple of x) and (x $\neq$ y) } (ii) R_2 = {(x, y) | x + y = 6 }

(2) For the following relations on X, determine if each relation is reflexive, symmetric, antisymmetric, and/or transitive. A relation could have more than one such property mentioned above. In that case state all properties. Explain your answers.
Let P = {1, 2, 3, 4, 5, 6, 7, ... } (set of all positive integers).
R_1 = {(x, y) | x $\geq$ y, where x, y $\in$ P},
R_2 = {(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)},
R_3 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)},
R_4 = {(x, y) | the positive difference |x - y| is evenly divisible by 4}.

(3) Let X = {1, 2, 3}. List a relation R on X (with at least 4 pairs) which is symmetric and transitive but not reflexive. Explain your answer.

(4) Let X = {1, 2, 3, 4}. Let R_1 = {(1, 2), (1, 3), (4, 2)} and R_2 = {(2, 3), (2, 4)}.
(i) List all elements/pairs of (R_1 o R_2) $\cap$ (R_2 o R_1). Show all steps.
(ii) List all pairs for $(R_1 \cup R_2)^2$.
(iii) Obtain transitive closure of ($R_1 \cup R_2$).

(5) If each of the following statements is true, explain why. If false, give a counter example. Assume that R and S are some relations on a nonempty set A.

(i) If R is transitive and S is transitive then R $\cup$ S is transitive.
(ii) If R is transitive and S is transitive then R $\cap$ S is transitive.
(iii) If R is symmetric then R^c (converse of R) is symmetric.
(iv) If R and S both are antisymmetric then so is R $\cup$ S.

Let r, s, and t denote the closure operators as discussed in class.

(v) The relation r(s(t(R))) is always an equivalence relation for any R.
(vi) The relation t(s(r(R))) is always an equivalence relation for any R.

(6) For X = {1, 2, 3, 4, 5}, determine if each of the following relations is an equivalence relation. If yes, explain your answer and give equivalence classes. If no, explain why.
(i) R_1 = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3) (3, 1), (3, 2), (3, 3), (4, 4), (5, 5) }
(ii) R_2 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (3, 1), (3, 3), (4, 1), (4, 4), (5, 5)}