CSE 213 Fall 2007

Homework 4

HW-4 is due in class on Friday, Nov 16, at the beginning.

Exam-2: Monday, Nov 19, whole class hour. **Topics:** Binary relations, Equivalence, Closures, Countability, Partial Orders, (Skip Lattices, Product and Filing order), Chains, Total order, Lex and Standard order for strings, Well founded order and its application to proofs by induction, Standard induction, Constructing well founded orders.

Correction: While defining partial order, we talked about reflexive and irreflexive partial orders. In class, I mentioned that only reflexive partial order can be a total order. That was not correct. Both $<$ and $\leq$ are total orders on N. This was later clarified in class. I have included this correction/note for those students who were not in class that day.

(1) Each of the following is a relation on {0, 1, 2, 3}. State and explain if any of them is a partial order.

R_1 = {(0, 0), (0, 2), (1, 0), (1, 3), (2, 2), (3, 0), (3, 1)}
R_2 = {(0, 0), (1, 0), (1, 2), (1, 3), (2, 0), (3, 2), (3, 0)}

(2) Let S be the set of strings over {a, b}. Define a relation R on S as follows. For strings u and v, u R v iff length(u) $\leq$ length(v). Is R a partial order? Prove or give a counter example.

(3) Define relations on the set Z of integers (+ and -) as follows. Check if they are partial orders. Prove or give a counter example.
(i) For all m, n in Z, mR_1n iff every prime factor of m is a prime factor of n.
(ii) For all m, n in Z, mR_2n iff m+n is even.

(4) Consider the divides relation on A = {1, 2, 4, 8, ..., 2^n}, where n is a nonnegative integer. Prove that this relation is a total order on A. First prove that divides is a partial order on A, for some n.

(5) Consider the divides relation on A = {1, 2, 4, 5, 10, 15, 20}. Here $A \subset N$. We say x is related to y, if x divides y. Draw the Hasse diagram for this relation on A. What are its greatest, least, maximal and minimal elements? Find two different chains of length 3. Length of a chain is number of elements on it - 1. What is the least upper bound for A? What is the greatest lower bound for A?

(6) Let A = {a, b, c}. Describe all partial order relations on A such that ‘a’ is a maximal element. You may use Hasse diagrams or list them as sets of ordered pairs.

(7) In lecture notes, we defined lexicographic order for strings and stated two theorems. Prove them. (i) The relation $<_L$ is a partial order. (ii) If $(\Sigma, <)$ is a totally ordered set then so is $(\Sigma^*, <_L)$.

(8) Consider the following strings defined over {a, b}. Order them lexicographically and also using standard order. Strings are: ab, aab, abb, ba, bba, abbba, aaab, abba, aaa, bbb.

(9) Prove using well founded induction that Ackerman’s function as given in notes is defined for all ordered pairs of integers from N.

(10) Q9 from page 252 of the text: Answer to this question is yes (given in the book). For this homework, you are supposed to prove it. So show that $N \times N$ is well founded with respect to the new relation. If you prove that there are no infinite descending chains/sequences, it should suffice.

(11) Prove using induction the next three statements.

(a) $1.3 + 2.4 + 3.5 + ... + n(n+2) = (n)(n+1)(2n+7)/6$
(b) For $n > 4$, show that $n^2 < 2^n$
(c) $1 + 2 + 4 + 8 + 16 + ... + 2^{n-1} = 2^n - 1$