CSE 213 Fall 2007

Homework 5

HW5: Due in class on Friday, Dec 14, at the beginning.
Final Exam: Friday Dec 21, 11am-1:30pm, Room will be announced later.

• For predicate logic, see pages 397-413 of the text. These pages are easy to read.

• Study questions given on pages 414 and 415 of the text [including Q16(a, b,c d)]. Many have solutions on the web. Most Q16 exercises have solutions in the book.

• For equivalence of quantified formulas read pages 417-420 of the text. Also see exercises Q7 and Q8 on page 431. Solutions are on the web.

(1) Using truth table, check if these formulas are logically equivalent. Give truth table for each.
(a) $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$
(b) $(p \leftrightarrow q) \leftrightarrow r$ and $p \leftrightarrow (q \leftrightarrow r))$

(2) Using identities for propositional logic, prove logical equivalence of these. No truth table here.
(a) $\neg (p \leftrightarrow q)$ and $p \leftrightarrow \neg q$
(b) $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$

(3) Using Quine's method check if the following formulas are tautology, contradiction or contingency.
(a) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$
(b) $[(p \rightarrow q) \rightarrow (r \rightarrow s)] \leftrightarrow [(p \rightarrow r) \rightarrow (q \rightarrow s)]$
(c) $\neg [(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)]$

(4) Is the following propositional formula satisfiable? Explain your answer (no truth table).
$(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$

(5) Let N(x) be a predicate 'x has visited North Dakota', where x is a student in CSE 213. Assume that domain is all students in CSE 213.
(a) Write quantified formulas for these.
(i) Some student has visited North Dakota. (ii) Not every student has visited North Dakota.
(b) Translate these in English. (i) $\neg\exists x N(x)$ (ii) $\forall x\, N(x)$

(6) Let C(x) mean that student x has a cat, and let D(x) mean that student x has a dog. Also let P(x) mean that student x has a parrot. Here x is a student in CSE 213. Assume that domain is all students in CSE 213.
(a) Write quantified formulas for the following.
(i) All students in CSE 213 either have a dog or a cat but not both.
(ii) Some student in CSE 213 does not have any of the above three pets.
(b) Translate these in English. (i) $\forall x(C(x) \vee D(x) \rightarrow P(x))$ (ii) $\neg \exists x\, (C(x) \wedge D(x) \wedge P(x))$

(7) Let Q(x) be the predicate $(x+1) > 2x$. Determine truth values if domain is the set Z.
(i) $Q(-1)$ (ii) $Q(1)$ (iii) $\forall x Q(x)$ (iv) $\forall x\, \neg Q(x)$ (v) $\neg \exists x Q(x)$

(8) Let $P(x)$ be some predicate and the domain of x is the set {-5, -3, -1, 1, 3}. Express the following quantified formulas using propositional formulas.(Use only AND, OR, NOT, and Implication etc.) Note that if $P(x)$ is a predicate then $P(1)$ is a proposition, because 1 is in the domain of x.
(i) $\exists x P(x)$ (ii) $\forall x P(x)$ (iii) $\forall x((x \text{ not equal to } 1) \dashrightarrow P(x))$ (iv) $\exists x((x > 0) \wedge P(x))$

(9) Q10(b) page 415 of the text. To answer this, study answer to Q10(a). For Q10(b), the domain is {a, b}. First decide how many possible interpretations are there for the wff W?

(10) Find a model for:
(i) $[\exists x p(x) \wedge \exists x q(x)] \dashrightarrow \exists x[p(x) \wedge q(x)]$ (ii) $\exists y \forall x[p(x,f(x)) \dashrightarrow p(x, y)]$

(11) find a countermodel for:
(i) $\exists x p(x) \dashrightarrow \forall x p(x)$ (ii) $[\exists x p(x) \wedge \exists x q(x)] \dashrightarrow \exists x[p(x) \wedge q(x)]$

(12) Give a predicate logic formula F_1 (without using the equality predicate) such that F_1 is not valid, but is true for any interpretation whose domain is a singleton set. Note that F_1 must use **exactly** one variable and at least one quantifier.

(13) Let EVEN(x) be a predicate which is TRUE if x is even otherwise it is FALSE. First state if each of the formula is true/false. Next negate each formula and simplify further using logical equivalence rules (and De Morgan's Laws), such that negation is applied to only to a predicate. Write the simplified negated formula in English as well. Domain is N (set of natural numbers).
(i) $\forall m \exists n \text{ EVEN}(m+n)$
(ii) $\forall m \forall n[\text{ EVEN}(m+n) \dashrightarrow \text{EVEN}(m) \vee \text{EVEN}(n)]$

(14) (i) Show that $(\exists x \forall y\ p(x, y) \dashrightarrow \forall y \exists x\ p(x, y))$ is a valid formula.
(ii) Show that $\forall y \exists x\ p(x, y)$ is not logically equivalent to $\exists x \forall y\ p(x, y)$. Come up with an interpretation that makes one of them true and the other false.