CSE213 Fall2007

Solution: Homework 5

(1) (a) NO (b) YES. use truth table.

(2) (a) L.H.S. $\equiv \neg(p \leftrightarrow q)$
$\equiv \neg((p \rightarrow q) \wedge (q \rightarrow p))$
$\equiv \neg((\neg p \vee q) \wedge (\neg q \vee p))$
$\equiv \neg(\neg p \vee q) \vee \neg(\neg q \vee p)$
$\equiv (p \wedge \neg q) \vee (q \wedge \neg p))$
R.H.S. $\equiv p \leftrightarrow \neg q)$
$\equiv (p \rightarrow \neg q) \wedge (\neg q \rightarrow p))$
$\equiv (\neg p \vee \neg q) \wedge (q \vee p)$
$\equiv ((\neg p \vee \neg q) \wedge q) \vee ((\neg p \vee \neg q) \wedge p)$
$\equiv ((\neg p \wedge q) \vee (\neg q \wedge q)) \vee ((\neg p \wedge p) \vee (\neg q \wedge p))$
$\equiv ((\neg p \wedge q) \vee false) \vee (false \vee (\neg q \wedge p))$
$\equiv (\neg p \wedge q) \vee (\neg q \wedge p)$
$\equiv (p \wedge \neg q) \vee (q \wedge \neg p))$
So, L.H.S. $\equiv$ R.H.S.
(b)L.H.S. $\equiv \neg p \rightarrow (q \rightarrow r)$
$\equiv p \vee (q \rightarrow r)$
$\equiv p \vee \neg q \vee r$
R.H.S. $\equiv q \rightarrow (p \vee r)$
$\equiv \neg q \vee (p \vee r)$
$\equiv p \vee \neg q \vee r$
So, L.H.S. $\equiv$ R.H.S.

(3) (a) Tautology (b) Contingency (c) Contradiction
Solution of (b)
$W \equiv [(p \rightarrow q) \rightarrow (r \rightarrow s)] \leftrightarrow [(p \rightarrow r) \rightarrow (q \rightarrow s)]$
$W(p/false) \equiv [(F \rightarrow q) \rightarrow (r \rightarrow s)] \leftrightarrow [(F \rightarrow r) \rightarrow (q \rightarrow s)].$
$\equiv [T \rightarrow (r \rightarrow s)] \leftrightarrow [T \rightarrow (q \rightarrow s)].$
$\equiv (r \rightarrow s) \leftrightarrow (q \rightarrow s).$
Now, Let $X \equiv (r \rightarrow s) \leftrightarrow (q \rightarrow s).$
$X(s/true) \equiv (r \rightarrow T) \leftrightarrow (q \rightarrow T).$
$\equiv T \leftrightarrow T.$
$\equiv T.$
$X(s/false) \equiv (r \rightarrow F) \leftrightarrow (q \rightarrow F).$
$\equiv \neg r \leftrightarrow \neg q.$
When r is false and q is true, this is false. so, the statement is a contingency.

(4) It's satisfiable. For example, look at p and q in every clause. If we make p=true and q=false then the formula evaluates to true and we don't need to look at r and s.

(5) (a) (i)$\exists_x N(x)$ (ii) $\neg\forall_x N(x)$
(b) (i) No student has visited North Dakota. (ii) Every student has visited North Dakota.

(6) (a) (i) $\forall_x((C(x) \wedge \neg D(x)) \vee (\neg C(x) \wedge D(x)))$
(ii)$\exists_x(\neg C(x) \wedge \neg D(x) \wedge \neg P(x))$
(b) (i) For every student if he/she has a cat or dog then he/she has a parrot.
(ii) No student has all three pets.

(7)(i) True (ii) False (iii) False (iv) False (v) False

(8) (i) $P(-5) \vee P(-3) \vee P(-1) \vee P(1) \vee P(3)$
$\exists_x$ becomes OR.
(ii) $P(-5) \wedge P(-3) \wedge P(-1) \wedge P(1) \wedge P(3)$
$\forall_x$ becomes AND.
(iii) $P(-5) \wedge P(-3) \wedge P(-1) \wedge P(3)$
When x is 1, (x not equal to 1) $\rightarrow P(x)$ is True. So, it can be dropped from AND(ii).
(iv) $P(1) \vee P(3)$
When $x > 0$ is false, $(x > 0) \wedge P(x)$ is false. It can be dropped from OR(i).

(9) There are 4 interpretations. (i) $p(a) = True, p(b) = True$. (ii) $p(a) = False, p(b) = False$. (iii) $p(a) = True, p(b) = False$. (iv) $p(a) = False, p(b) = True$.
Truth value of W in those interpretations: (i) True, (ii) True, (iii) False, (iv) False.

(10) (i) Let domain, D is the set of positive integers.
$p(x) =$ x is divisible by 2 and $q(x) =$ x is divisible by 3.
$\exists_x p(x)$ is true, $x = 2$.
$\exists_x q(x)$ is true, $x = 3$.
$\exists_x(p(x) \wedge q(x))$ is true, $x = 6$.
So, the implication is true and this is a model.
(ii) Let domain, D is the set of positive integers.
$f(1) = 1$. when x is 1.
$f(x) = x - 1$. otherwise.
and $p(x, y) = (x >= y)$.
Now when y is 1, we get,
$\forall_x p(x, x - 1) \rightarrow p(x, 1)$ which is always true.
So, the implication is true and this is a model.

(11) (i) $p(x) =$ x is odd. where domain is the set of positive integers.
$\exists_x p(x)$ is true since there are some integers which are odd e.g., $x = 1$.
$\forall_x p(x)$ is false since all integers are not odd. e.g.,$x = 2$.
(ii) $p(x) =$ x is odd and $q(x) =$ x is even, where domain is the set of positive integers.

(12) $\exists_x p(x) \rightarrow \forall_x p(x)$ is not valid when $p(x) =$ x is odd. But it is true for any singleton set D=a. Since p(a) can be either true or false, the implication becomes for singleton set $T \rightarrow T$ or $F \rightarrow F$. So, for singleton set this is true.

(13) (i) $\forall_m \exists_n EVEN(m + n)$ is True. If m is odd, find odd n and if m is even find even n, and then (m+n) will be even.
$\neg\forall_m \exists_n EVEN(m + n)$
$\equiv \exists_m \forall_n \neg EVEN(m + n)$
There exists m, such that for all n, (m+n) is not even. This statement is false because there is no such m. This is consistent, so the original statement is true.

(ii) $\forall_m \forall_n [EVEN(m+n) \rightarrow EVEN(m) \vee EVEN(n)]$ is False.
$\neg\forall_m \forall_n [EVEN(m+n) \rightarrow EVEN(m) \vee EVEN(n)]$.
$\equiv \exists_m \exists_n \neg(\neg EVEN(m+n) \vee EVEN(m) \vee EVEN(n))$.
$\equiv \exists_m \exists_n (EVEN(m+n) \wedge \neg EVEN(m) \wedge \neg EVEN(n))$.
There is some m and some n such that (m+n) is even and m is not even and n is not even. This is true for odd m and n. So, the original statement is false.

(14)(i) If the L.H.S is false, then the whole implication becomes true. Now consider any interpretation that makes L.H.S true. So, there is some x, which makes p(x,y) true for all y. If we choose the same x, then for all y $\exists_x p(x,y)$ would also be true. So, the given statement is valid.

(ii) Let, domain, $D = \{1, 2, 3\}$ and p(x,y) is $x = y$.

$\forall_y \exists_x p(x,y)$ means for every y there exist some x such that x equals y. This is true because you can choose $x = y$ for every y.

Now,$\exists_x \forall_y p(x,y)$ means there is some x such that for all y, x is equal to y. So, here x is fixed first and there is no such x. So, it is false.

These two statements are not logically equivalent since for this interpretation one of them is true and other is false.