CSE213 Fall2007

Solution: Homework 1

(1) $A - B = A \cap B^c$.

A-B is the set of elements in A that are not in B. B^c is the complement of B with respect to some universal set U. So, $B^c = U - B$. It contains all elements of U except B. So, its intersection with set A will give only those elements in A that are not in B. Hence $A - B = A \cap B^c$.

(2) $A - (B \cap C) = (A - B) \cup (A - C)$

$x \in A - (B \cap C)$ iff $x \in A$ and $x \notin B \cap C$

iff $x \in A$, and either $x \notin B$ or $x \notin C$

iff either ($x \in A$ and $x \notin B$) or ($x \in A$ and $x \notin C$)

iff $x \in (A - B) \cup (A - C)$

Hence, $A - (B \cap C) = (A - B) \cup (A - C)$

(3) (i) and (iv) are partitions of A.

(4) (i) {a,b,c,d}

(ii) {{a},{b},{c},{d}}

(iii) {{a},{b,c,d}}, {{b},{a,c,d}}, {{c},{a,b,d}}, {{d},{a,b,c}},{{a,b},{c,d}},{{a,c},{b,d}},{{a,d},{b,c}}

(5) Total number of subsets of a set with k elements = 2^k. Number of subsets excluding both 1 and 2 is 2^{k-2}. Now if we include 1 to every subset then total number of subsets will remain same, 2^{k-2}.

So, if $k = 1$ then 1; if $k > 1$, then 2^{k-2}.

(6) A truth table for k variables has 2^k entries. Since each k variable function can map to either True or False, then total number of k-variable Boolean functions is 2^{2^k}

(7) $f(x) = (-1)^x \cdot 3x$; where $x \in N$.

(8)(i) aaa, aab, aba, abb, baa, bab, bba, bbb.

(ii) a, b, ab, ba, aa, aba, baa, abaa

(iii) prefixes: ababbba, ababbb, ababb, abab, aba, ab , a, $\wedge$

suffixes: $\wedge$, a, ba, bba, bbba, abbba, babbba, ababbba

(9)Basis: $bb \in S$

Induction: If $x \in S$ and $w \in X^*$, then $xw \in S$.

(10)Basis: $0 \in S$

Induction: If $x \in S$ and $x/14 \in Z$, then $x + 14, x - 14 \in S$.

If $x \in S$ and $x/23 \in Z$, then $x + 23, x - 23 \in S$.