CSE 213 Fall '07: Midterm 2 Solutions

1a) Every function from a set A to a se B is also a relation from A to B (Yes)
1b) Every reflexive relation on a set A, is also a function from A to A itself (No, because it canhave more pairs)
1c) The power se of every finite set is countable (Yes)
1d) The poset (Z,<) is not well-founded with respect to < (Yes)
1e) Every totally ordered finite set has the least element and the greatest element (Yes)

2) N={0,1,2,3,…}
R = {(x,y)| the positive difference |x-y| is odd}
2a)Any 4 elements of R: (1,2),(1,4),(4,1),(2,1),(5,2),…
2b) R is not an equivalence relation
2c) R is not reflexive $(1,1) \notin R$
R is not transitive, for instance $(1,4), (4,5) \in R$ but $(1,5) \notin R$

3) X={1,2,3} and R={(1,1),(1,3),(2,1),(2,2),(3,1),(3,3)}be a relation on X
3a) This relation is not a partial order because it is not anti-symmetric (1,3), (3,1) are in the relation and $1 \neq 3$. Also, it is not transitive; (2,1) and (1,3) are in R but (2,3) isn't.
3b) To make it partial order, remove (1,3)
3c) Remove symmetric pair so that it makes R anti-symmetric and also makes it satisfy transitivity by default.

4) Give a relation R on N such that for relation R, it's the diagonal set D and its complement set are both countably infinite.
4a) R={(1,1),(3,3),(5,5),(7,7), …}
R= {(x,x)| x is odd numbers}
4b) Diagonal set D={1,3,5,…}
Its complement= {0,2,4,6,8,…}
4c) Set D has all odd numbers. It is infinite and it is subset of a countably in infinite set N. The complement of D has all even numbers.

5)
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5a)      8    12    15
         |     |     |
         |     6     |
          \  /   \  /
            2      3
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5b) Greatest: None   Maximal: 8,12, 15
Least: None   Minimal: 2,3

5c) LUB =120 because it is the least common multiple 8,12,15

6) Standard Order:
00, 11, 012, 021, 210, 0221, 1112, 2021, 10201

7) Suppose we define a relation R on NxN such that (a,b) is related to (c,d) as follows. Hear a, b, c, d are in N
(a,b)R(c,d) if a>c or [a=c] and [b>d]
7a) (4,2) is not related to (4,3)
7b) (0,0) is successor of all pairs. It is also the maximal element(greatest)
We have descending chains that are infinitely long; therefore, R is not well founded

8) n > 0
$1.2+2.2^2+3.2^3+\ldots+n.2^n = (n-1)2^{n+1}+2$

Base case: n=1
LHS= 1.2 = 2
RHS= $(0).2^2+2 = 2$
LHS=RHS
Base case holds

Induction Hypothesis:
Assume that the above statement is true for arbitrary but fixed n
To prove it for n+1:
LHS for n+1
$= 1.2+2.2^2+3.2^3+\ldots+n.2^n +(n+1)2^{n+1}$
$= (n+1).2^{n+1}+ 2 + (n+1).2^{n+1}$
$=2^{n+1}(n-1+n+1) + 2$
$=2^{n+1}(2n) + 2$
$=n,2^{n+2} + 2$
$=(n+1-1)(2)^{n+2} + 2$ --- RHS QED