

## SOLUTION: QUIZ 3

(1)(a) YES. (b)  $P \rightarrow Q$  is a tautology. By contrapositive we get  $\neg Q \rightarrow \neg P$ . So,  $\neg Q \rightarrow \neg P$  is also a tautology. So,  $\neg P$  is a logical consequence of  $\neg Q$ .

OR, another explanation:

$$P \rightarrow Q \equiv \neg P \vee Q \equiv Q \vee \neg P \equiv \neg Q \rightarrow \neg P.$$

(2)(a) YES. (b) Since  $P \rightarrow Q$  is a contradiction, it's always false. It's only false when P is true and Q is false. So, P is a tautology and Q is a contradiction. Since Q is a contradiction,  $\neg Q$  is a tautology. Therefore, P and  $\neg Q$  are logically equivalent.

$$(3) W = ((p \wedge q) \rightarrow r) \rightarrow ((p \rightarrow r) \wedge (q \rightarrow r)).$$

$$W(p/\text{false}) \equiv ((\text{false} \wedge q) \rightarrow r) \rightarrow ((\text{false} \rightarrow r) \wedge (q \rightarrow r)).$$

$$\equiv (\text{false} \rightarrow r) \rightarrow ((\text{true}) \wedge (q \rightarrow r)).$$

$$\equiv \text{true} \rightarrow ((\text{true}) \wedge (q \rightarrow r)).$$

$$\equiv \text{true} \rightarrow (q \rightarrow r).$$

$$\equiv q \rightarrow r.$$

Now, Let  $X \equiv q \rightarrow r$ .

$$X(q/\text{true}) \equiv \text{true} \rightarrow r \equiv r.$$

When  $r=\text{true}$  this is true and when  $r=\text{false}$  this is false. So, the logic sentence is a contingency.