Language accepted by a FSA

- Let a FSA $A = (I, S, s0, F, N)$.

  $N : S \times I \to S$ can be extended to $N : S \times I^* \to S$ to deal with strings.

  Example:

  Let a state $s0$ and a string $abba$, $N(s0, abba) = N(N(N(N(s0, a), b), b), a)$.

- If L is the set of strings that a FSA A accepts, we say that L is the language of A and we write $L(A) = L$.

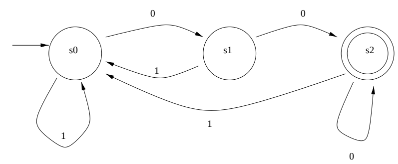
- Generally, we conjecture what is the language L and we prove that $L(A) = L$ by proving that $L \subseteq L(A)$ and $L(A) \subseteq L$.

  To prove that $L \subseteq L(A)$, we need to prove that for all word $w \in L$, then $w \in L(A)$ i.e. $N(s0, w) \in F$.

  To prove that $L(A) \subseteq L$, we need to prove that for all word $w \in L(A)$, then $w \in L$. So we need to prove that if $N(s0, w) \in F$, then $w \in L$.

Example

- What is the language recognized by this automaton A?



- 10 is not accepted
  10100 is accepted
  00 is accepted
  110010 is not accepted
  1101000 is accepted

- Ideas?

  If we are in state $s0$ and if we read a sequence of 1, we stay in $s0$. (We can prove this property by induction – see below).

  If we are in state $s2$ and if we read a sequence of 0, we stay in $s2$. (We can prove this property by induction).

  If we read 2 zeros we go in the final state $s2$ but then if we read 1, we go out of the final state $s2$.

- Conjecture:

  Any string w that ends with 00 is accepted by A. w is a string of length greater than 2.

  $L = \{w \in I^* \mid w = w'.00\}$

- We prove using the structure of w that if $w \in L$ i.e. $w = w'.00$ then w is accepted by A. (We prove that $L \subseteq L(A)$).

  w is a string of length n that ends with 00. So $w = w'.00$. The size of w is greater than 2.

  After the first $n - 2$ symbols of w have been input, A is in one of its three states: $s0$, $s1$ and $s2$. From any of these three states, input of the symbols 00 in will result in A moving to the accept state s2 ($N(s0, 00) = s2$ and $N(s1, 00) = s2$ and $N(s2, 00) = s2$). Hence, any string that ends in 00 is accepted by M.

- We prove by induction that $N(s0, 0^n) = s0$ ($n \geq 0$).

  0^n means a sequence of 0 or more 0. O^0 represent the empty word denoted ϵ.

  - *Basis case:* n = 0
    $N(s0, \epsilon) = s0$
  - *Induction hypothesis:* $N(s0, 0^k) = s0$ for an arbitrary and fixed $k \geq 0$.
  - We prove that: $N(s0, 0^{k+1}) = s0$.
    $N(s0, 0^{k+1}) = N(N(s0, 0^k), 0) = s0$.
  - Conclusion: $N(s0, 0^n) = s0$ for all $n \geq 0$.

Example

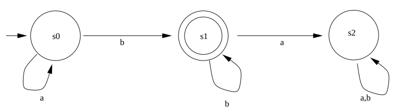
- Design an automaton that recognizes
  $L = \{a^p b^q \mid p \geq 0 \; and \; q > 0\}$.

- What are the words of L?

  $a \notin L$ $(q > 0)$
  $aba \notin L$
  $b \in L$
  $ab \in L$
  $aaaaaabbbb \in L$

  Words that begin with 0, 1 or several a's and terminate with b's (at least 1).

- Automaton A - Transitions diagram:



- We prove that $L = L(A)$.

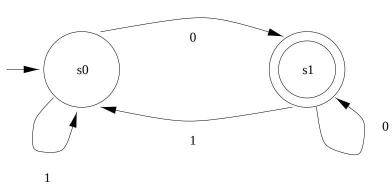
  Sketch of the proof.

  – $L \subseteq L(A)$
  Let $w \in L$. We prove that $N(s0, w) = s1$.
  To do that we prove:
  – $\forall p \geq 0, N(s0, a^p) = s0$ (by induction on p).
  – $\forall q \geq 1, N(s0, a^p b^q) = s1$ (by induction on q).

  – $L(A) \subseteq L$
  Let w such that $N(s0, w) = s1$. We prove that $w \in L$.
  To do that we prove:
  – $\forall w, N(s0, w) = s0 \Rightarrow w = a^p$ (by induction on the length of w).
  – $\forall w$ (of length greater than 1), $N(s0, w) = s1 \Rightarrow \exists p \geq 0, \exists q > 0, w = a^p b^q$ (by induction on the length of w).

Example

- What is the language recognized by this automaton?



- 11 is not accepted
  10101 is not accepted
  1 is not accepted
  110010 is accepted
  1101000 is accepted

- Ideas?

  If we are in state $s0$ and if we read a sequence of 1, we stay in $s0$. (We can prove this property by induction).

  If we are in state $s1$ and if we read a sequence of 0, we stay in $s1$. (We can prove this property by induction).

  If we are in state $s0$ and if we read a 0 we go to state $s1$.

  If we are in state $s1$ and if we read a 1 we go to state $s0$.

- Conjecture:

  Any string w that ends with 0 is accepted by A. w is a string of length greater than 1. A recognizes even numbers.

  $L = \{w \in I^* \mid w = w'.0\}$

- We can prove that $L = L(A)$.

The man, the wolf, the goat and the cabbage

http://www.ecs.soton.ac.uk/ uun/CM219/HTML/
sld032.htm

Slides 32 to 41