Practice Problems Solutions

1. Prove the following Boolean expression using algebra.

A. X'Y' + X'Y + XY = X' + Y
= (X'Y+ X'Y') + (X' Y + XY) replication of term X'Y
= X'(Y + Y') + Y(X + X')
= X' + Y

B. A'B + B'C' + AB + B'C = 1
= (A'B+ AB) + (B'C'+ B'C)
= B(A + A') + B'(C + C')
= B + B'
= 1

C. Y + X'Z + XY' = X + Y + Z
= Y + X Y' + X' Z
= Y(1 + X) + XY' + X'Z
= (Y + X)(Y + Y') + X'Z
= Y + X + X'Z
= Y + (X + X')(X + Z)
= X + Y + Z

D. X'Y' + Y'Z + XZ + XY + YZ' = X'Y' + XZ + YZ'
= X' Y' + Y' Z(X + X') + XZ + XY + Y Z'
= X' Y' + X Y' Z + X' Y' Z + XZ + XY + Y Z'
= X' Y' (1 + Z) + X Y' Z +XZ + XY + Y Z'
= X' Y' + XZ(1 + Y') + XY + Y Z'
= X' Y' + XZ + XY (Z + Z')+ Y Z'
= X' Y' + XZ + XY Z +Y Z' (1 + X)
= X' Y' + XZ(1 + Y) + Y Z'
= X' Y' + XZ + Y Z'

E. AB' + A'C'D'+ A'B'D + A'B'CD' = B' + A'C'D'

= AB'(C+C')(D+D') + A'C'D'(B+B') + A'B'D(C+C') + A'B'CD'
= AB'CD + AB'C'D + AB'CD' + AB'C'D' + A'BC'D' + A'B'C'D' + A'B'CD + A'B'C'D + A'B'CD'
= AB'CD + AB'C'D + AB'CD' + AB'C'D' + A'B'CD + A'B'C'D + A'B'CD' + A'B'C'D'+ A'BC'D' + A'B'C'D'
= B'(A+A')(C+C')(D+D') + A'C'D'(B+B')
= B'+A'C'D'

Alternate approach:
AB' + A'C'D' + A'B'D + A'B'CD'
= B' (A + A'C'D' + A'D + A'CD') + A'C'D'
(replicate A'C'D') (A'C'D hides B')
= B'(A + A' (C'D' + D + CD') + A'C'D'
= B'(A + A'(D+ D'(C'+C)) + A'C'D'
= B'(A + A'(D+D'(1)) + A'C'D'
= B'(A + A'(D+D')) + A'C'D'
= B'(A + A'(1)) + A'C'D'
= B'(A + A') + A'C'D'
= B'(1) + A'C'D'
= B'+A'C'D'

F. XZ + WY'Z'+W'YZ'+WX'Z' =
XZ + WY'Z' + WXY' + W'XY + X'YZ'
= XZ(W+W')(Y+Y') + WY'Z'(X+X') + W'YZ'(X+X') + WX'Z'(Y+Y')

= ***XZ(W+W')(Y+Y'):*** **WXYZ + W'XYZ + WXY'Z + W'XY'Z**
WY'Z'(X+X'): **WXY'Z' + WX'Y'Z'**
W'YZ'(X+X'): **W'XYZ' + W'X'YZ'**
WX'Z'(Y+Y'): **WX'YZ' + WX'Y'Z'**

= WXYZ + W’XYZ + WXY’Z + W’XY’Z + WXY’Z’ + W’XYZ’ + WX’YZ’ + W’X’YZ’ + WX’Y’Z’

= **XZ(W+W’)(Y+Y’):* WXYZ + W’XYZ + WXY’Z + W’XY’Z
****WY’Z’(X+X’):* WXY’Z’ + WX’Y’Z’**
***WXY’(Z+Z’):* WXY’Z + WXY’Z’**
***W’XY(Z+Z’):* W’XYZ + W’XYZ’**
***X’YZ’(W+W’):* WX’YZ’ + W’X’YZ’**

= XZ(W+W’)(Y+Y’) + WY’Z’(X+X’) + WXY’(Z+Z’) + W’XY(Z+Z’) + X’YZ’(W+W’)

G. CD + AB’ + AC + A’C’ + A’B + C’D’ =
(A’ + B’ + C + D’)(A + B + C’ + D)

= CD(A+A’)(B+B’) + C’D’(A+A’)(B+B’) + AB’(C+C’)(D+D’) + A’B(C+C’)(D+D’) + AC(B+B’)(D+D’)+ A’C’(B+B’)(D+D’)

= *CD(A+A’)(B+B’):* ABCD + A’BCD + AB’CD + A’B’CD
***C’D’(A+A’)(B+B’):* ABC’D’ + A’BC’D’+ AB’C’D’ + A’B’C’D’**
***AB’(C+C’)(D+D’):* AB’CD + AB’C’D + AB’CD’ + AB’C’D’**
***A’B(C+C’)(D+D’):* A’BCD + A’BC’D + A’BCD’ + A’BC’D’**
***AC(B+B’)(D+D’):* ABCD + AB’CD + ABCD’ + AB’CD’**
***A’C’(B+B’)(D+D’):* A’BC’D + A’B’C’D + A’BC’D’ + A’B’C’D’**

= ABCD + A’BCD + AB’CD + + ABC’D + ABCD’ + A’B’CD + AB’C’D + ABC’D’ + + A’BCD’ + AB’CD’ + A’BCD’ + A’BC’D + AB’C’D’ + A’B’C’D + A’BC’D’

= **A’B(C+C’)(D+D’):* A’BCD + A’BC’D + A’BCD’ + A’BC’D’
****A’C’(B+B’)(D+D’):* A’BC’D + A’B’C’D + A’BC’D’ + A’B’C’D’**
***A’D(B+B’)(C+C’):* A’BCD + A’B’CD+ A’BC’D + AB’C’D**
****AB’(C+C’)(D+D’):* AB’CD + AB’C’D + AB’CD’ + AB’C’D’**
***B’C’(A+A’)(D+D’):* AB’C’D + A’B’C’D + AB’C’D’ + A’B’C’D’**
***B’D(A+A’)(C+C’):* AB’CD + A’B’CD + AB’C’D + A’B’C’D**
****AC(B+B’)(D+D’):* ABCD + AB’CD + ABCD’ + AB’CD’**
***BC (A+A’)(D+D’):* ABCD + A’BCD + ABCD’ + A’BCD’**
****CD(A+A’)(B+B’):* ABCD + A’BCD + AB’CD + A’B’CD**
***AD’(B+B’)(C+C’):* ABCD’ + AB’CD’ + ABC’D’ + AB’C’D’**
***BD’(A+A’)(C+C’):* ABCD’ + A’BCD’ + ABC’D’ + A’BC’D’**
****C’D’(A+A’)(B+B’):* ABC’D’ + A’BC’D’ AB’C’D’ + A’B’C’D’**

= A’B(C+C’)(D+D’) + A’C’(B+B’)(D+D’) + A’D(B+B’)(C+C’) + AB’(C+C’)(D+D’) + B’C’(A+A’)(D+D’)+ B’D(A+A’)(C+C’) + AC(B+B’)(D+D’) + BC (A+A’)(D+D’)+ CD(A+A’)(B+B’) + AD’(B+B’)(C+C’) + BD’(A+A’)(C+C’) + C’D’(A+A’)(B+B’)

= A’B + A’C’ + A’D + AB’ + B’C’+ B’D + AC + BC + CD + AD’ + BD’ + C’D’
= AA’ + A’B + A’C’ + A’D + AB’ + BB’ +B’C’ + B’D + AC + BC + CC’ + CD + AD’ + BD’ + C’D’ + DD’

= (A’ +B’ + C + D’) (A + B + C’ + D)

Note that the * denotes lines which are the same as in step 2. All other terms are repeats. Also, note that the only term missing is A’B’CD’ this implies that the truth table has only 1 zero (0010). The function can be represented as $\prod M(2)$.

2. Simplify the following Boolean expressions to the minimum number of literals (total number of appearances of all variables, eg. AB+C’ has 3 literals).
   A. ABC + ABC’ + A’B **= B**
   B. (A + B)’(A’ + B’) **= A’B’**
   C. A’BC + AC = **AC + BC**
   D. BC + B(AD + AD’) **= B(C + A)**
   E. (A + B’ + AB’)(AB + A’C +BC) **= AB + A’B’C**

3. Reduce the following expressions to the indicated number of literals (total number of appearances of all variables, eg. AB+C' has 3 literals).

A. X'Y' + XYZ + X'Y to 3 literals
= X' + XYZ = (X' + XY)(X' + Z)
= (X' + X)(X' + Y)(X' + Z) = (X' + Y)(X' + Z)
= X'+YZ

B. X + Y(Z + (X + Z)') to 2 literals
= X + Y(Z + X' Z') = X + YZ + X'YZ' = X + (YZ + X')(YZ + YZ')
= X + Y(X' + YZ) = X + X'Y + YZ = (X + X')(X + Y) + YZ
= X + Y + YZ
= X +Y

C. W'X(Z' + Y'Z) + X(W + W'YZ) to 1 literals
= **W'XZ' + W'XY'Z + WX + W'XYZ = WX + W'XZ' + W'XZ**
= WX + W'X = X

D. ((A + B) + A'B')(C'D' + CD) + A'C' to 4 literals
= ABC'D' + ABCD + A'B'C'D' + A'B'CD + A' + C'
= A'(1+ B'C'D' + B'CD) + C'(1 + ABD') + ABCD
= A'(1 + BCD) + C'+ ABCD = A' + A'BCD + C' + ABCD
= A' + C' + (A+A')BCD
= A' + C'(1 + BD) + BCD = A' + C' + BC'D + BCD
= A' + C'+ (C'+C)(BD)
= A' + C'+ BD

4. Find the complement of the following expressions

A. AB' + A'B **= (A' +B)(A+B')**
B. (V'W +X)Y +Z' **= ((V+W')X'+Y')Z**
C. WX(Y'Z+YZ') + W'X'(Y' + Z)(Y +Z')
= [W'+X'+(Y+Z')(Y'+Z)][W+X+YZ'+Y'Z]
D. (A +B' + C)(A'B' + C)(A + B'C') **= A'BC'+(A+B)C'+A'(B+C)**

5. Obtain the truth tables for the following expressions

A. **Z** = (XY + Z)(Y + XZ)

X	Y	Z	**Z**
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

B. **Z** = (A' + B)(B' + C)

X	Y	Z	**Z**
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

C. **Z** = WXY' + WXZ' + WXZ +YZ'

W	X	Y	Z	**Z**
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

6. Convert the following truth table to switching expression (Boolean Algebra), and simplify the expression as much as possible

X	Y	Z	**E**
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

E = X + Y'Z

X	Y	Z	**G**
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

G = Y

7. Using DeMorgan's theorem, express the function
   F = ABC + A'C' + A'B'
   a. with only OR and complement operators
   b. with only AND and complement operators

Solution:
a. F = (A' + B' + C')' + (A+C)' + (A+B)' = (A'+B'+C')' + (A + (B' + C')')'
b. F = (ABC)'(A'C')'(A'B')' or [(ABC)' (A'(BC)')']'

Minterms & Maxterms

8. Write the truth table for the following functions, and express the functions as sum-of-minterms and product-of-maxterms
   c. (XY + Z)(Y + XZ)
   a. (A' + B)(B' + C)
   b. WXY' + WXZ' + WXZ + YZ'

Solution:

a.

X	Y	Z	a
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

sum-of-minterms: X'YZ + XY'Z + XYZ' + XYZ
product-of-maxterms: (X + Y + Z) (X + Y + Z')
(X + Y' +Z)(X' + Y + Z)

b.

A	B	C	b
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

sum-of-minterms:A'B'C' + A'B'C + A'BC + ABC
product-of-maxterms: (A + B'+ C) (A' + B + C)
(A' + B + C')(A' + B' + C)

c.

W	X	Y	Z	c
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

sum-of-minterms: W'X'YZ' + W'XYZ' + WX'YZ' + WXY'Z' + WXY'Z + WXYZ' + WXYZ
product-of-maxterms:(W+X+Y+Z) (W+X+Y+Z') (W+X+Y'+Z')(W+X'+Y+Z)(W+X'+Y+Z') (W+X'+Y'+Z')(W'+X+Y+Z) (W'+X+Y+Z')(W'+X+Y'+Z')

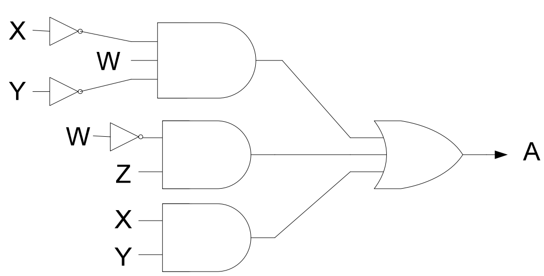
9. Convert the following expressions into sum-of-products (minterms) and product-of-sums (maxterms)
   d. (AB +C)(B + C'D)
   = AB + ABC'D + BC + CC'D = AB + ABC'D + BC = AB(1+C'D) + BC
   = AB +BC (SOP)
   = B(A+C) = (B+B)(A+C) (POS)

   e. X' + X(X + Y')(Y + Z')
   = (X'+X) (X' + (X + Y'))(Y+Z') = (X' + X + Y')(X' + Y + Z')
   = X' + Y + Z' (SOP & POS)

   f. (A + BC' + CD)(B' + EF)
   = (A + BC' + CD)(B'+E)(B'+F)
   = (A + B + C) (A + B + D) (A + C'+D)(B'+E) (B'+F) (POS)

   =A(B'+EF) + BC'(B'+EF) + CD(B'+EF)
   = AB' + AEF + BC'EF + B'CD + CDEF (SOP)

10. Convert the following gate diagrams into (1) switching expression, (2) truth table, (3) sum-of-products, and (4) product-of-sums



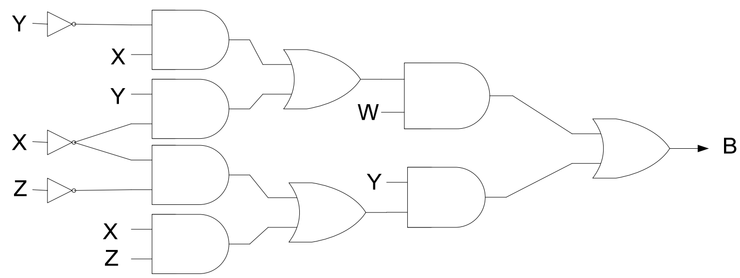
(1) switching expression : WX'Y' + W'Z + XY
(2)

W	X	Y	Z	A
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1

0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

(3) sum-of-products: W’X’Y’Z+ W’X’YZ +W’XY’Z + W’XYZ’+ W’XYZ +WX’Y’Z’ + WX’Y’Z+ WXYZ’ + WXYZ

(4) product-of-sums: (W+X+Y+Z)(W+X+Y’+Z)(W+X’+Y+Z)(W’+X+Y’+Z)(W’+X+Y’+Z’)(W’+X’+Y+Z)(W’+X’+Y+Z’)



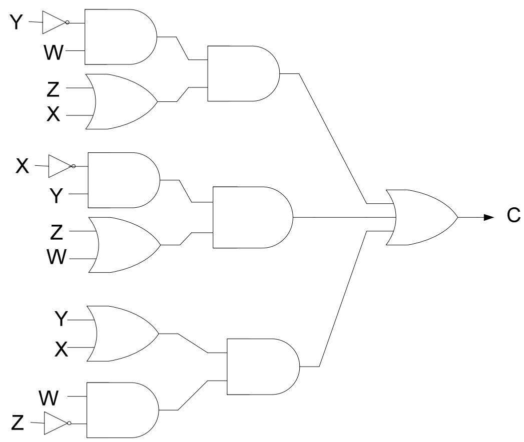
(1) switching expression : W(XY’+X’Y) + Y(XZ+X’Z’)

(2)

W	X	Y	Z	B
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

(3) sum-of-products: W’X’YZ’+ W’XYZ +WX’YZ’ + WX’YZ+ WXY’Z’ +WXY’Z + WXYZ

(4) product-of-sums: (W+X+Y+Z)(W+X+Y+Z’)(W+X+Y’+Z’)(W+X’+Y+Z)(W+X’+Y+Z’)(W+X’+Y’+Z)(W’+X+Y+Z)(W’+X+Y+Z’)(W’+X’+Y’+Z)



(1) switching expression : WY'(X+Z) +X'Y(W+Z) + WZ'(X+Y)

(2)

W	X	Y	Z	C
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

(3) sum-of-products: W'X'YZ+ WX'Y'Z +WX'YZ' + WX'YZ+ WXY'Z' +WXY'Z + WXYZ'+WXYZ

(4) product-of-sums: (W+X+Y+Z)(W+X+Y+Z')(W+X+Y'+Z)(W+X'+Y+Z)
(W+X'+Y+Z')(W+X'+Y'+Z)(W+X'+Y'+Z')(W'+X+Y+Z)(W'+X'+Y'+Z')

11. Simplify/write the following expressions in (1) sum-of-products and (2) product-of-sums forms

g. AC' + B'D + A'CD + ABCD
= AC' + B'D(1 + AC) + A'CD(B+B')A+ ABCD
= AC' + B'D + AB'CD + A'BCD + A'B'CD + ABCD
= AC' + B'D + CD(AB' + A'B + A'B' + AB)
= CD + AC'+B'D (SOP)
= (C'+D)(A+D)(A+B'+C) (POS)

h. (A' + B + D')(A + B' + C')(A' + B + D')(B + C' + D')
= A'C'+B'D' +AD' (SOP)
= (C'+D')(A'+D')(A+B'+C') (POS)

i. (A' + B' + D)(A' + D')(A + B + D')(A + B' + C + D))
= A'BD+B'D' +A'BC or A'BD+B'D' +A'CD' (SOP)
= (A'+B')(B+D')(B'+C+D) (POS)

j. *F*(A,B,C,D) = ∑*m*(2,3,5,7,8,10,12,13)
= A'B'CD'+ A'B'CD+ A'BC'D +A'BCD +AB'C'D'+AB'CD'+ABC'D'+ABC'D
= AB'D'+ABC'+A'BD+A'B'C+B'CD' (there are multiple answers) (SOP)
= (A+B'+D)(B+C+D')(A+B+C)(A'+C'+D')(A'+B'+C') (there are multiple answers) (POS)

k. $F(W,X,Y,Z) = \prod M(2,10,13)$

= Y'Z' + W'X + X'Z + XY **(SOP)**

= (W+X+Y'+Z)(X'+Y+Z') **(POS)**

12. For the Boolean functions given in the following truth table:

X	Y	Z	E	F	G
0	0	0	1	0	1
0	0	1	1	0	1
0	1	0	1	1	1
0	1	1	0	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	1	1
1	1	1	0	1	0

l. List the minterms and maxterms of each function

E = $\sum m$(0,1,2,5) **F = $\sum m$(2,3,6,7)** **G = $\sum m$(0,1,2,4,6)**

E = $\prod M$(3,4,6,7) **F = $\prod M$(0,1,4,5)** **G = $\prod M$(3,5,7)**

m. List the maxterms of E', F', and G'

E' = $\prod M$ (0,1,2,5) **F' = $\prod M$ (2,3,6,7)** **G' = $\prod M$ (0,1,2,4,6)**

E' = $\sum m$ (3,4,6,7) **F' = $\sum m$ (0,1,4,5)** **G' = $\sum m$ (3,5,7)**

n. Write the truth tables for E + F and EF

X	Y	Z	E	F	E+F	EF
0	0	0	1	0	1	0
0	0	1	1	0	1	0
0	1	0	1	1	1	1
0	1	1	0	1	1	0
1	0	0	0	0	0	0
1	0	1	1	0	1	0
1	1	0	0	1	1	0
1	1	1	0	1	1	0

o. List the minterms of E + F and EF

E+F = $\sum m$(0,1,2,3,5,6,7) =X'Y'Z+X'Y'Z+X'YZ'+X'YZ+XY'Z+XYZ'+XYZ

EF =$\sum m$(2) = X'YZ'

p. Express E, F and G in sum-of-products

E = X'Y'Z' + X'Y'Z + X'YZ' + XY'Z

F = X'YZ' + X'YZ + XYZ' + XYZ

G = X'Y'Z' + X'Y'Z + X'YZ' + XY'Z' + XYZ'

q. Express E, F and G in products-of-sums

E = (X+Y'+Z')(X' + Y + Z) (X'+Y'+Z)(X' + Y' + Z')

F = (X+Y+Z)(X+Y+Z')(X'+Y+Z)(X'+Y+Z')

G = (X+Y'+Z')(X'+Y+Z')(X'+Y'+Z')

r. Simplify E, F and G to expressions with a minimum number of literals (sum-of-products).

E = X'Y'Z' + X'Y'Z + X'YZ' + XY'Z
= X'Z'(Y'+Y) + Y'Z(X+X')
= Y'Z + X'Z'

F = X'YZ' + X'YZ + XYZ' + XYZ
= YZ'(X+X') + YZ(X'+X)
= Y(Z' + Z) = Y

G= X'Y'Z' + X'Y'Z + X'YZ' + XY'Z' + XYZ'
=X'Y'(Z+Z') + Z'(X'Y +XY' +XY + X'Y') **duplicate X'Y'Z'**
= X'Y' + Z'