Homework #3 Solutions

Quiz in Recitations on

Monday, November 16, and Wednesday, November 18

Problem 1: Using the postulates of Boolean algebra, prove the following formulae:
a) $x'y'z' + x'y'z + x'yz + xy'z + xyz = x'y' + z$
b) $ABC' + A'C'D + AB'C' + BC'D + A'D = AC' + A'D$
c) $wxy + w'xy + x'(zw + zy') + z(x'w' + y'x) = xy + z$

Solution:

a) $x'y'z' + x'y'z + x'yz + xy'z + xyz = x'y' + z$

$$
\begin{aligned}
x'y'z' + x'y'z(1+1) + x'yz + xy'z + xyz &= x'y' + z \\
x'y'z' + x'y'z + x'y'z + x'yz + xy'z + xyz &= x'y' + z \\
x'y'(z' + z) + z(x'y' + x'y + xy' + xy) &= x'y' + z \\
x'y' + z((x + x')y' + (x + x')y) &= x'y' + z \\
x'y' + z(y' + y) &= x'y' + z \\
x'y' + z &= x'y' + z
\end{aligned}
$$

b) $ABC' + A'C'D + AB'C' + BC'D + A'D = AC' + A'D$

$$
\begin{aligned}
ABC' + A'C'D + AB'C' + BC'D + A'D &= AC' + A'D \\
ABC' + AB'C' + BC'D + A'D(C' + 1) &= AC' + A'D \\
ABC' + AB'C' + BC'D + A'D &= AC' + A'D \\
AC'(B + B') + BC'D + A'D &= AC' + A'D \\
AC' + BC'D(A + A') + A'D &= AC' + A'D \\
AC' + ABC'D + A'BC'D + A'D &= AC' + A'D \\
AC'(1 + BD) + A'D(1 + BC') &= AC' + A'D \\
AC' + A'D &= AC' + A'D
\end{aligned}
$$

c) $wxy + w'xy + x'(zw + zy') + z(x'w' + y'x) = xy + z$

$$\begin{aligned}
(w + w')xy + x'(zw + zy') + z(x'w' + y'x) &= xy + z \\
xy + z(x'w + x'y') + z(x'w' + y'x) &= xy + z \\
xy + z(x'w + x'y' + x'w' + y'x) &= xy + z \\
xy + z(x'(w + w') + y'(x + x')) &= xy + z \\
xy + z(x' + y') &= xy + z \\
xy + z(xy)' &= xy + z \\
xy(1 + z) + z(xy)' &= xy + z \\
xy + xyz + z(xy)' &= xy + z \\
xy + z(xy + (xy)') &= xy + z \\
xy + z &= xy + z
\end{aligned}$$

Problem 2: Functions F, G, and H are defined in the following way:

$$F = A'C' + A'B'C$$
$$G = A'B' + A'C'$$
$$H = A'B'C' + A'C' + B'C$$

Which of the functions are equivalent?

Solution:

A	B	C	F	G	H
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	0	1
1	1	0	0	0	0
1	1	1	0	0	0

Functions F and G are equivalent.

By Algebra:

$$\begin{aligned}
F = A'C' + A'B'C &\quad ?= \quad A'B' + A'C' = G \\
F = A'C' + A'B'C &\quad ?= \quad A'B'(C + C') + A'C' = G \\
F = A'C' + A'B'C &\quad ?= \quad A'B'C + A'B'C' + A'C' = G \\
F = A'C' + A'B'C &\quad ?= \quad A'B'C + A'C'(B' + 1) = G \\
F = A'C' + A'B'C &\quad == \quad A'B'C + A'C' = G
\end{aligned}$$

$$\begin{aligned}
H = A'B'C' + A'C' + B'C &\quad ?= \quad A'C' + A'B'C = F \\
H = A'C'(B + 1) + (A' + A)B'C &\quad ?= \quad A'C' + A'B'C = F \\
H = A'C' + A'B'C + AB'C &\quad != \quad A'C' + A'B'C = F
\end{aligned}$$

Can not simplify anymore. To ensure that, you can't simplify anymore, the best way is to write out each side to minterms or maxterms and compare minterms.

Problem 3: Convert the following truth table to a switching expression (Boolean Algebra) and simplify the expression as much as possible.

x	y	z	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Solution:

x	y	z	F	minterm
0	0	0	1	x'y'z'
0	0	1	1	x'y'z
0	1	0	1	x'yz'
0	1	1	0	
1	0	0	0	
1	0	1	1	xy'z
1	1	0	0	
1	1	1	0	

$F = x'y'z' + x'y'z + x'yz' + xy'z$
$F = x'z'(y' + y) + y'z(x' + x)$
$F = x'z' + y'z$

Problem 4: Find all minterms and maxterms of the following functions:
a) $f = ((A' + B)' + C')' + DC' + AB'$
b) $g = x(y + w'z) + (w' + x' + z')'$

Solution:
a)
$f = ((A' + B)' + C')' + DC' + AB'$
$f = (A' + B)C + DC' + AB'$
$f = A'C + BC + DC' + AB'$

A	B	C	D	f
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Minterms: $\sum m = (1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15)$
Maxterms: $\prod M = (0, 4, 12)$

b)
$g = x(y + w'z) + (w' + x' + z')'$
$g = xy + w'xz + wxz$
$g = xy + (w' + w)xz$
$g = xy + xz$
$g = x(y + z)$

w	x	y	z	g
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Minterms: $\sum m = (5, 6, 7, 13, 14, 15)$
Maxterms: $\prod M = (0, 1, 2, 3, 4, 8, 9, 10, 11, 12)$

Problem 5: For each of the following functions, F & G:

x	y	z	F	G
0	0	0	0	1
0	0	1	1	1
0	1	0	0	0
0	1	1	0	0
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	0

a) Name the minterms ($\sum m(?)$) and maxterms ($\prod M(?)$) of f and g.
b) Give the boolean expressions in the sum of products form for f and g. Simplify.
c) Name the minterms ($\sum m(?)$) and maxterms ($\prod M(?)$) for f' and g'.
d) Give the boolean expressions for f' and g' in product of sums form for f and g. DON'T Simplify.

Solution:

a)

F: $\sum m = (1, 5, 6, 7)$ $\qquad \prod M = (0, 2, 3, 4)$
G: $\sum m = (0, 1, 4, 5, 6)$ $\qquad \prod M = (2, 3, 7)$

b)

$F = x'y'z + xy'z + xyz' + xyz$
$F = (x' + x)y'z + xy(z + z')$
$F = y'z + xy$

$G = x'y'z' + x'y'z + xy'z' + xy'z + xyz'$
$G = x'y'z' + x'y'z + xy'z + xy'z'(1 + 1) + xyz'$
$G = (x'y'z' + x'y'z + xy'z + xy'z') + (xy'z' + xyz')$
$G = y'(x'z' + x'z + xz' + xz) + xz'(y + y')$
$G = y'(x + x')(z + z') + xz'$
$G = y' + xz'$

c)

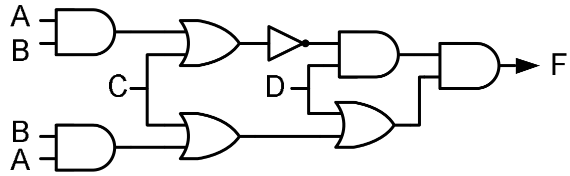
F: $\sum m = (0, 2, 3, 4)$ $\qquad \prod M = (1, 5, 6, 7)$
G: $\sum m = (2, 3, 7)$ $\qquad \prod M = (0, 1, 4, 5, 6)$

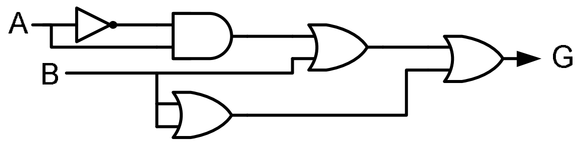
d)

$$F = (x + y + z')(x' + y + z')(x' + y' + z)(x' + y' + z')$$

$$G = (x + y + z)(x + y + z')(x' + y + z)(x' + y + z')(x' + y' + z)$$

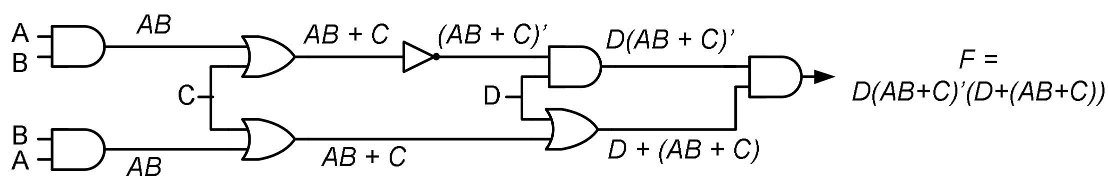
Problem 6: For the following diagrams, give boolean expressions. Simplify and redraw the system.

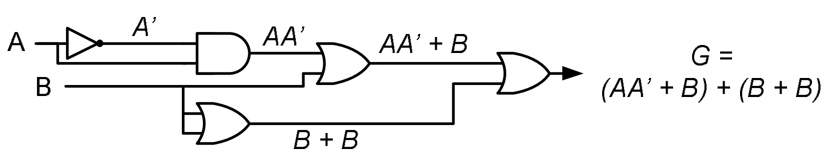




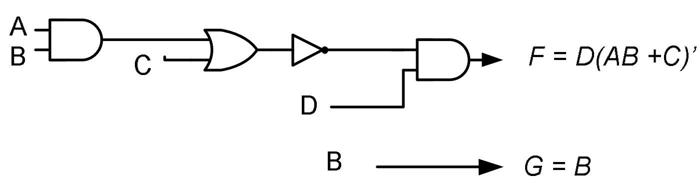
Solution:

Circuit Function:





Simplified Circuits:



Problem 7: Show that the operation (gates) # described by the following table is universal. Use, if needed, either constant 0 or constant 1.

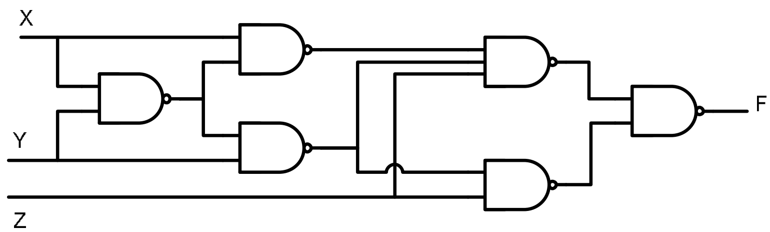
x	y	x#y
0	0	0
0	1	1
1	0	0
1	1	0

Solution:

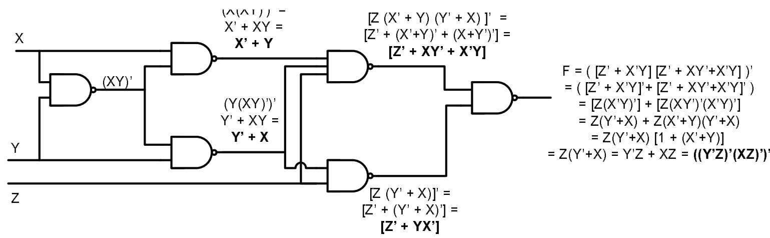
x#y = x'y
NOT Gate: set y = 1, x#1 = x'(1)
AND Gate: set x = x' x'#y = (x')'y = xy

{#} is a universal set.

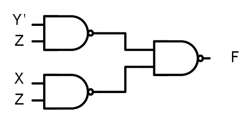
Problem 8: Obtain the truth table for the circuit shown. Give the proper minimal boolean expression for this circuit (either sum of products or products of sums). Draw an equivalent circuit for F with fewer NAND gates.



Solution:



$$F = ((Y'Z)'(XZ)')' \Leftrightarrow (Y'Z) + (XZ)$$



Problem 9: Implement the following expression using multiplexers: $f = x'_1 x'_2 + x'_3 x_2 x'_1 + x_1 x'_2$

Solution: $f = x'_2 (x'_1 + x_1) + x'_3 \, x_2 \, x'_1$

$f = x'_2 + x'_3 \, x_2 \, x'_1$

$f = x'_2 (1) + x_2 (x'_3 \, x'_1)$