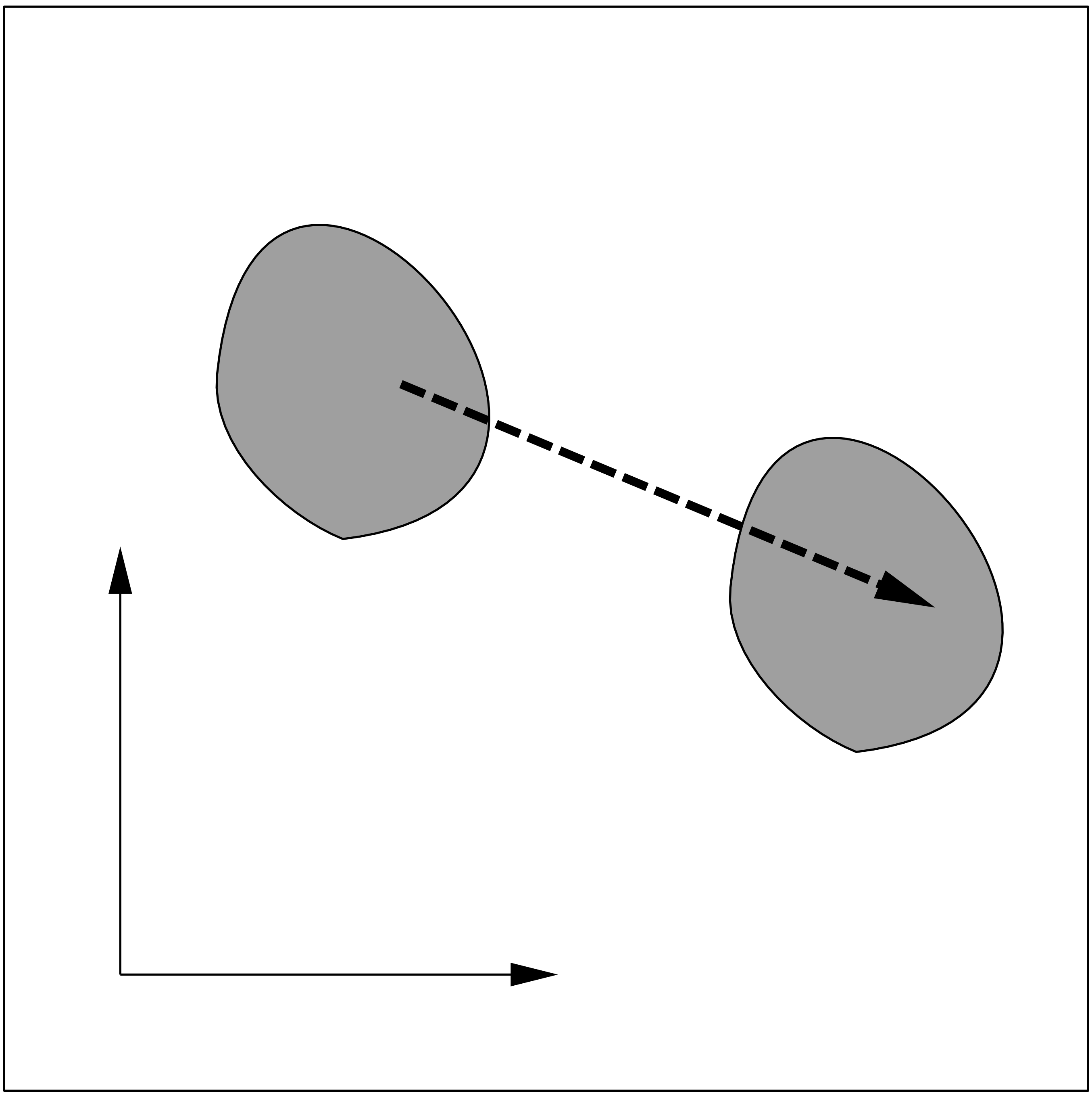
2D Translation



2D Translation

- **Current position**

$$\mathbf{p} = \begin{bmatrix} x \\ y \end{bmatrix}$$

- **New position (after translation)**

$$\mathbf{p}' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

- **Translation operation**

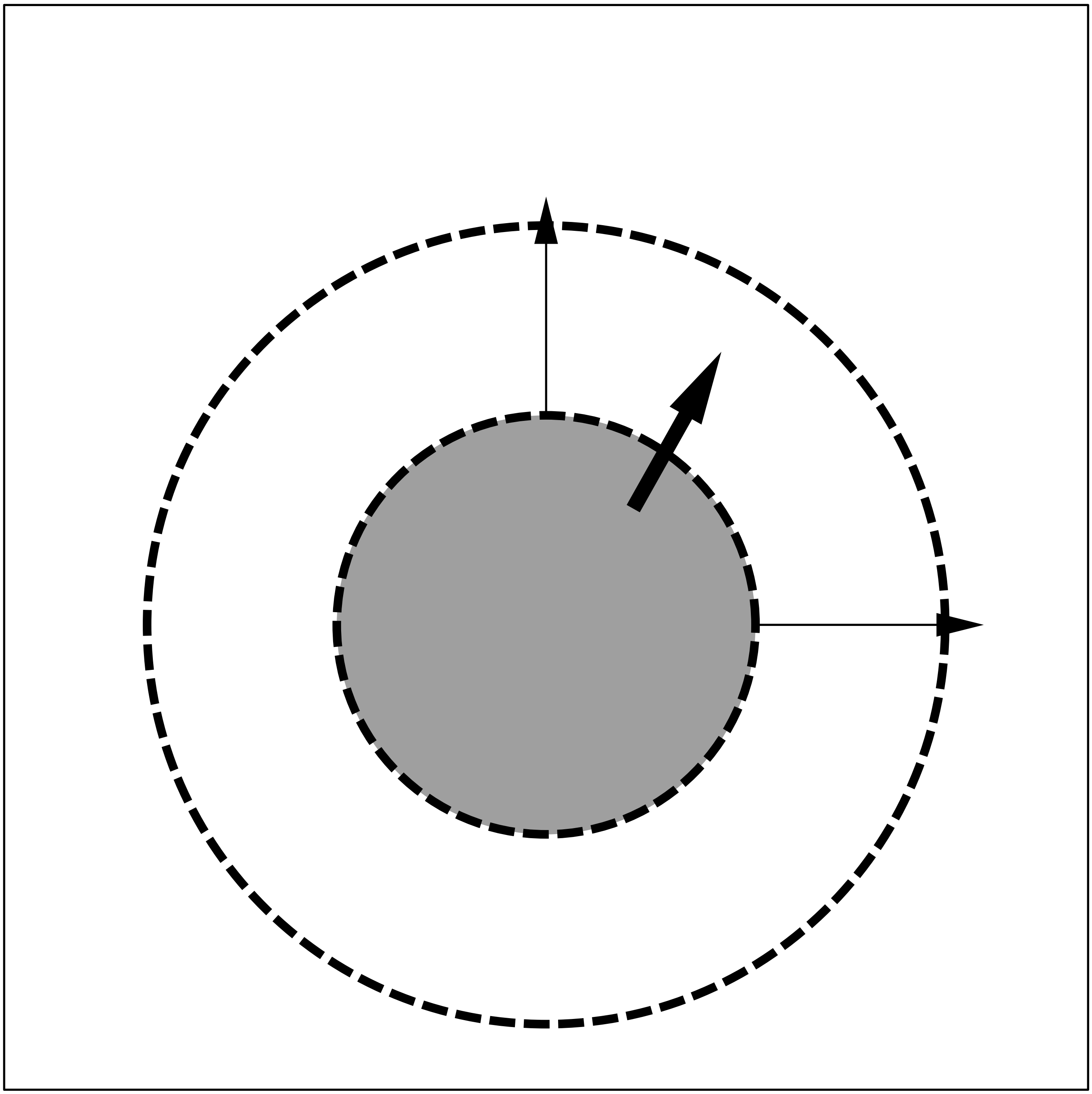
$$T(\delta x, \delta y) = \begin{bmatrix} \delta x \\ \delta y \end{bmatrix}$$

$$\mathbf{p} + T(\delta x, \delta y) = \mathbf{p}'$$

$$x' = x + \delta x$$

$$y' = y + \delta y$$

2D Scaling



2D Scaling

- **Current position**

$$\mathbf{p} = \begin{bmatrix} x \\ y \end{bmatrix}$$

- **New position**

$$\mathbf{p}' = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$$

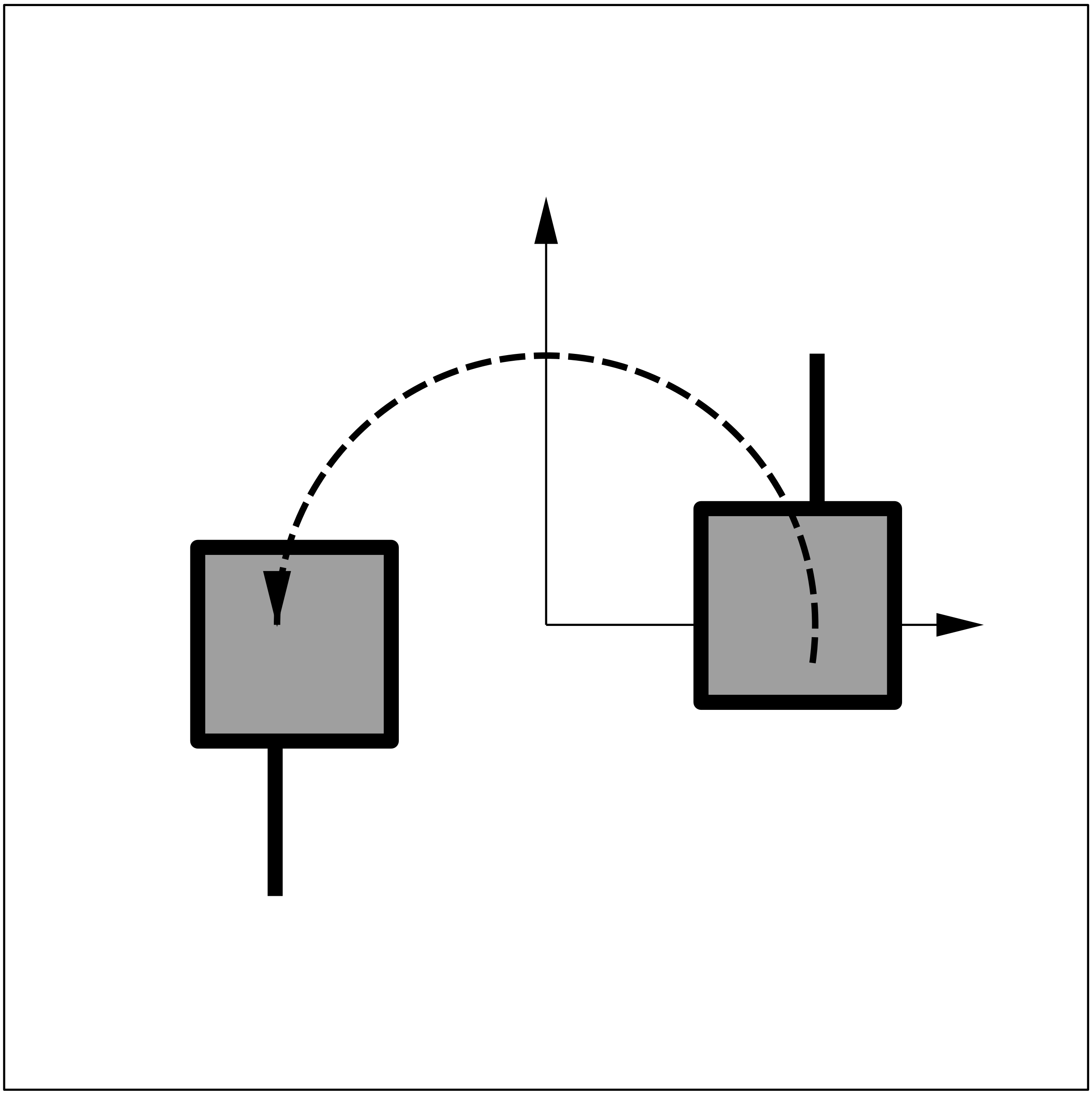
- **Scaling operation**

$$S(s_x, s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

$$S(s_x, s_y)\mathbf{p} = \mathbf{p}'$$

- **Matrix multiplication**

2D Rotation



2D Rotation

- **Current position**

$$\mathbf{p} = \begin{bmatrix} x \\ y \end{bmatrix}$$

- **New position**

$$\mathbf{p}' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

- **Rotation operation**

$$R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$R(\theta)\mathbf{p} = \mathbf{p}'$$

$$\mathbf{p}' = \begin{bmatrix} x\cos(\theta) - y\sin(\theta) \\ x\sin(\theta) + y\cos(\theta) \end{bmatrix}$$

- **Positive angles are "counter-clockwise"!**

- **Derivation of rotation**

$$x = r\cos(\theta_1)$$

$$y = r\sin(\theta_1)$$

- **Rotate θ_2**

$$x' = r\cos(\theta_1 + \theta_2)$$

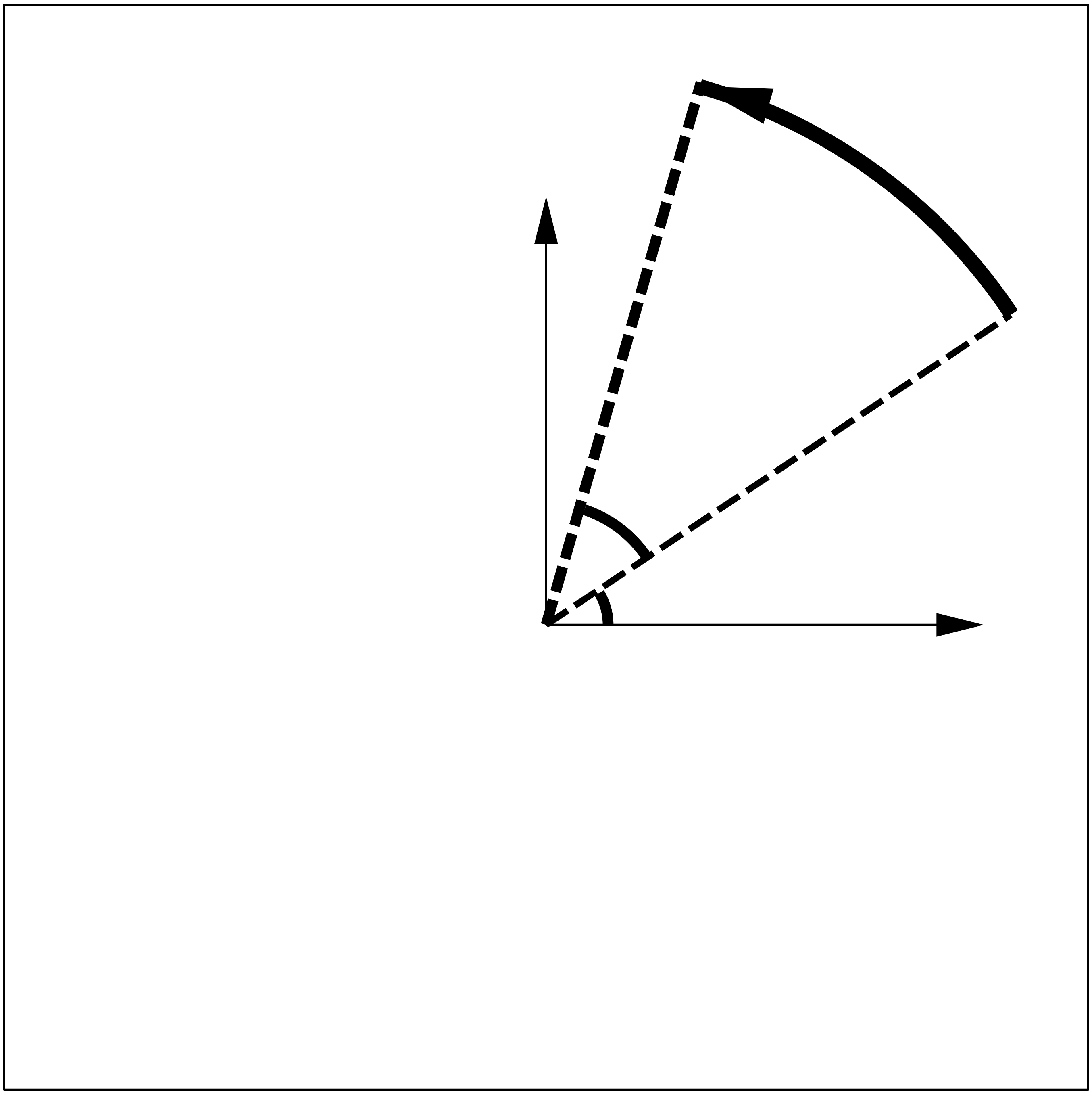
$$y' = r\sin(\theta_1 + \theta_2)$$

- **Observation (important results from trigonometry)!**

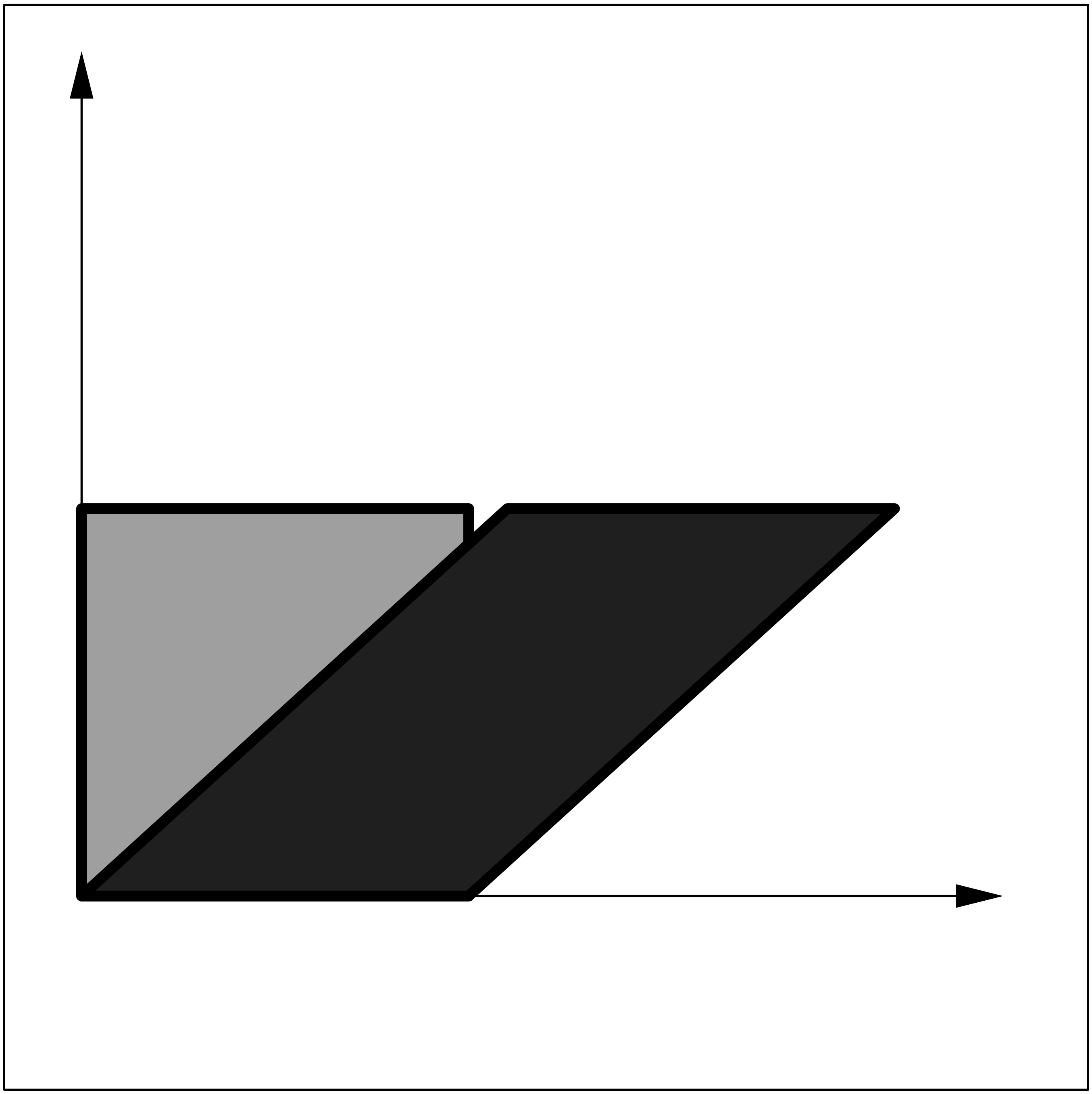
$$x' = r\cos(\theta_1)\cos(\theta_2) - r\sin(\theta_1)\sin(\theta_2)$$

$$y' = r\cos(\theta_1)\sin(\theta_2) + r\sin(\theta_1)\cos(\theta_2)$$

2D Rotation



2D Shear



2D Shear

- **Current position**

$$\mathbf{p} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{p}' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x + ay \\ y \end{bmatrix}$$

- **Shear operation**

$$Sh_x(a) = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{p}' = Sh_x(a)\mathbf{p}$$

- **Geometric meaning**

- **Shear operation along y axis**

$$Sh_y(b) = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}$$

$$\mathbf{p}' = Sh_y(b)\mathbf{p} = \begin{bmatrix} x \\ bx + y \end{bmatrix}$$

- **Geometric meaning !**
- **Consider more complicated cases!**
  Various examples are shown in the class!