3D Graphics Concepts

- 3D coordinate system
 - -x, y, and z
 - depth information
- Geometric modeling of various objects
 - point, line, polygon
 - curve, surface, solid
- Geometric transformation
- 3D Viewing
 - parallel projection
 - perspective projection
- Display methods of 3D objects
 - wireframe
 - shaded objects
 - visible object identification

- realistic rendering techniques
- 3D stereoscopic viewing

3D Transformation

• 3D translation

$$\mathbf{T}(\delta x, \delta y, \delta z) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

• 3D scaling

$$\mathbf{S}(a,b,c)\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}ax\\by\\cz\end{bmatrix}$$

• 3D rotation

$$\mathbf{R}(z,\theta) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \cos(\theta) - y \sin(\theta) \\ x \sin(\theta) + y \cos(\theta) \\ z \end{bmatrix}$$

- Note that, a positive rotation about z-axis is defined as one rotation from positive x-axis to positive y-axis
- We can similarly define rotations about

- the other two axes
- About x-axis:
- from positive y-axis to positive z-axis About y-axis:
- from positive z-axis to positive x-axis
- One example:
 - One scaling operation S(a, b, c) followed by a translation T(d, e, f)
 - Let's write the matrix formulation

$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} d \\ e \\ f \end{bmatrix}$$

- Our goal is to combine them into an integrated representation
- We use homogeneous representation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \Longleftrightarrow \begin{bmatrix} xh \\ yh \\ zh \\ h \end{bmatrix}$$

$$\left[egin{array}{c} x \ y \ z \ 1 \end{array}
ight]$$

Scaling operation in matrix form

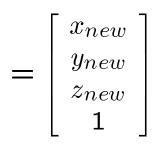
a	0	0	0
0	b	0	0
0	0	c	0
0	0	0	1
-			-

Translation operation in matrix form

Γ	1	0	0	d
	0	1	0	e
	0	0	1	f
	0	0	0	1]

The previous example (in matrix form)

$$\begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & e \\ 0 & 0 & 1 & f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Rotation

• W.r.t. z-axis

$$\mathbf{R}(z,\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0\\ \sin(\theta) & \cos(\theta) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix}$$

• W.r.t. x-axis

$$\mathbf{R}(x,\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

• W.r.t. y-axis

$$\mathbf{R}(y,\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Complex Operations

- Shearing operation
- Composition of transformations
- Inversion of transformations
- A series of transformations can be accumulated into a single transformation matrix
- One example
 Rotate along an axis (defined by x = 2 and y = 3)
 by -90⁰
- The operation consists of three steps
 (1) A = T(-2, -3, 0)
 (2) B = R(z, -90⁰)
 (3) C = T(2, 3, 0)
- The new object after this transformation is

$$Obj_{new} = \mathbf{C} \star \mathbf{B} \star \mathbf{A} \star Obj_{old}$$

• So, general form

$$\begin{bmatrix} x'\\y'\\z'\\1 \end{bmatrix} = \begin{bmatrix} X & X & X & t_x\\X & X & X & t_y\\X & X & X & t_z\\0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x\\y\\z\\1 \end{bmatrix}$$

- Composite transformations are non-commutative
- Transformations can be inverted