CSE328 Fundamentals of Computer Graphics: Theory, Algorithms, and Applications

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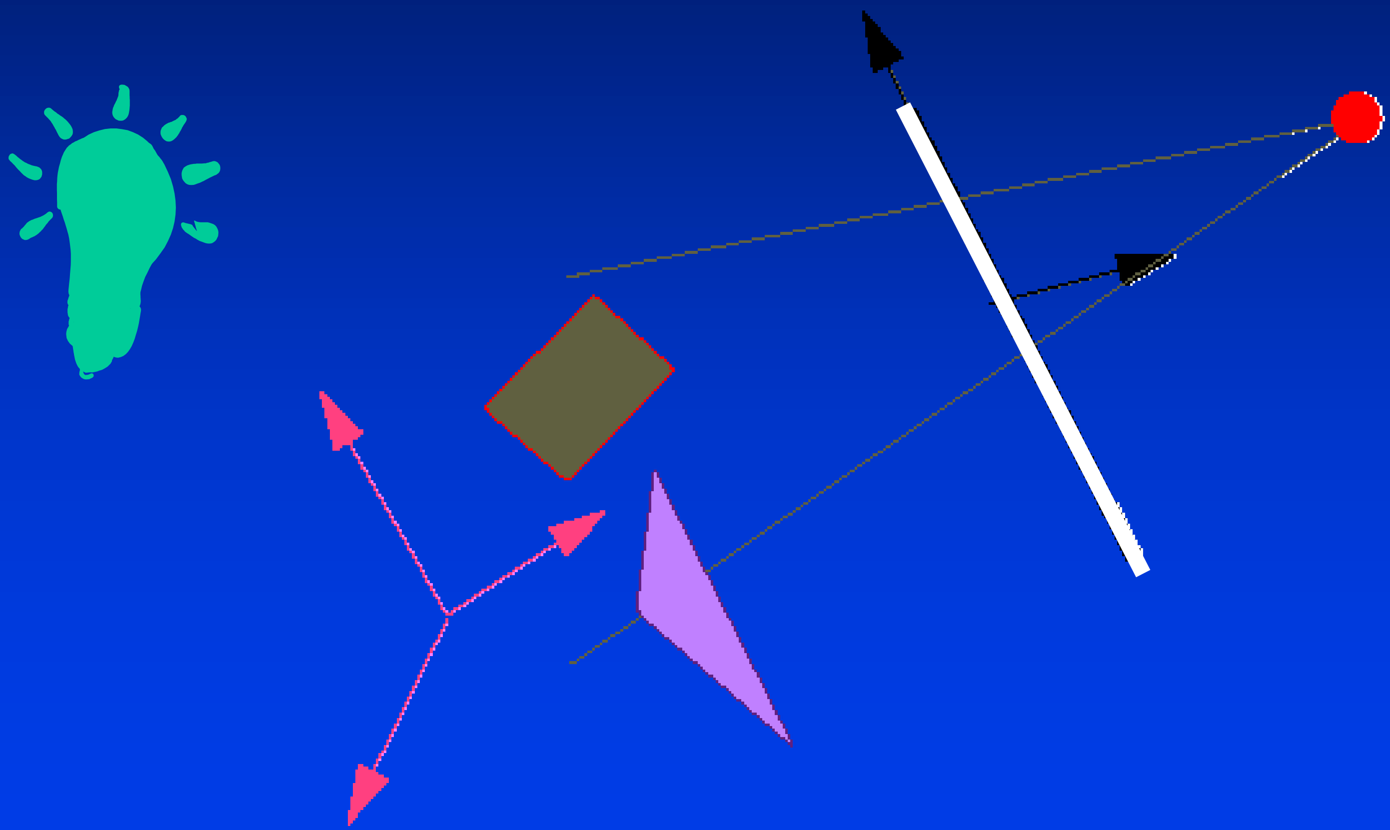
Global Illumination

- Global Illumination
  - A point is illuminated by more than light from local lights
  - It is illuminated by all the emitters and reflectors in the global scene
    - Ray Tracing
    - Radiosity

Ray Tracing

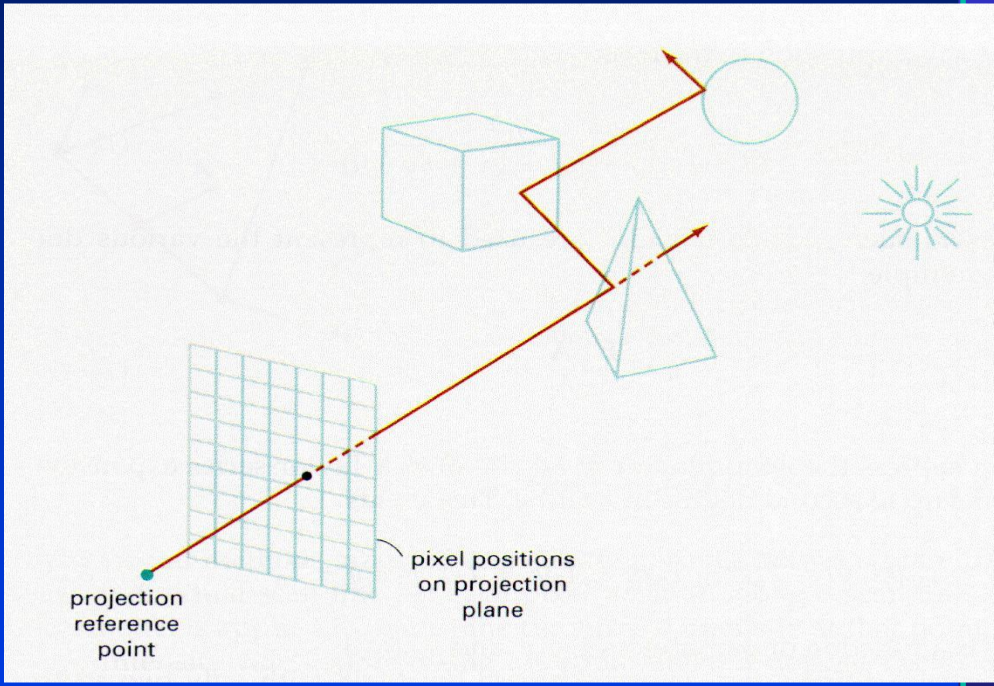


Ray Tracing Fundamentals

- Represent *specular* global lighting
- Trace light backward (usually) from the eye, through the pixel, and into the scene
- Recursively bounce off objects in the scene, accumulating a color for that pixel
- Final output is single image of the scene

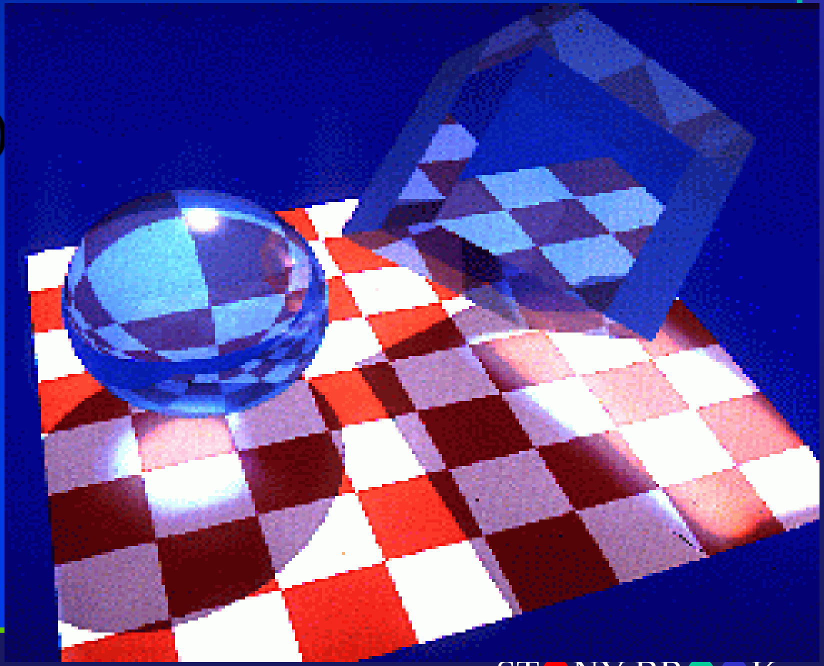
Recursive Ray Tracing

- Cast a ray from the viewer's eye through each pixel
- Compute intersection of this ray with objects from scene
- Closest intersecting object determines color



Recursive Ray Tracing

- For each ray cast from the eyepoint
  - If surface is struck
    - Cast ray to each light source (shadow ray)
    - Cast reflected ray (feeler ray)
    - Cast transmitted ray (feeler ray)
    - Perform Phong lighting on all incoming light
  - Note that, diffuse component of Phong lighting is not pushed through the system



Recursive Ray Tracing

- Computing all shadow and feeler rays is slow
  - Stop after fixed number of iterations
  - Stop when energy contributed is below threshold
- Most work is spent testing ray/plane intersections
  - Use bounding boxes to reduce comparisons
  - Use bounding volumes to group objects
  - Parallel computation (on shared-memory machines)

Recursive Ray Tracing

- Just a sampling method
  - We'd like to cast infinite rays and combine illumination results to generate pixel values
  - Instead, we use pixel locations to guide ray casting
- Problems?

Problems With Ray Tracing

- Aliasing
  - Supersampling
  - Stochastic sampling
- Works best on specular surfaces (not diffuse)
- For perfectly specular surfaces
  - Ray tracing == rendering equation (subject to aliasing)

Ray Tracing - Pros

- Simple idea and nice results
- Inter-object interaction possible
  - Shadows
  - Reflections
  - Refractions (light through glass, etc.)
- Based on real-world lighting

Ray Tracing - Cons

- Takes a long time
- Computation speed-ups are often highly scene-dependent
- Lighting effects tend to be abnormally sharp, without soft edges, unless more advanced techniques are used
- Hard to put into hardware

Supersampling - I

- Problem: each pixel of the display represents one single ray
  - Aliasing
  - Unnaturally sharp images
- Solution: send multiple rays through each "pixel" and average the returned colors together

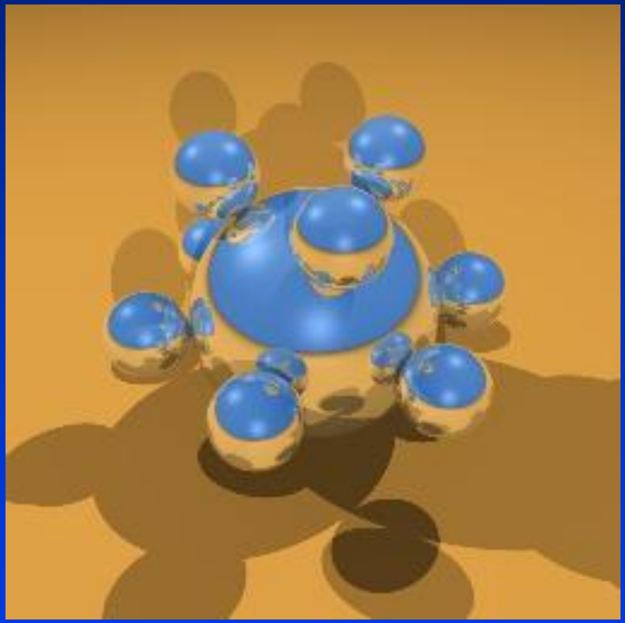
Supersampling - II

- Direct supersampling
  – Split each pixel into a grid and send rays through each grid point
- Adaptive supersampling
  – Split each pixel only if it's significantly different from its neighbors
- Jittering
  – Send rays through randomly selected points within the pixel

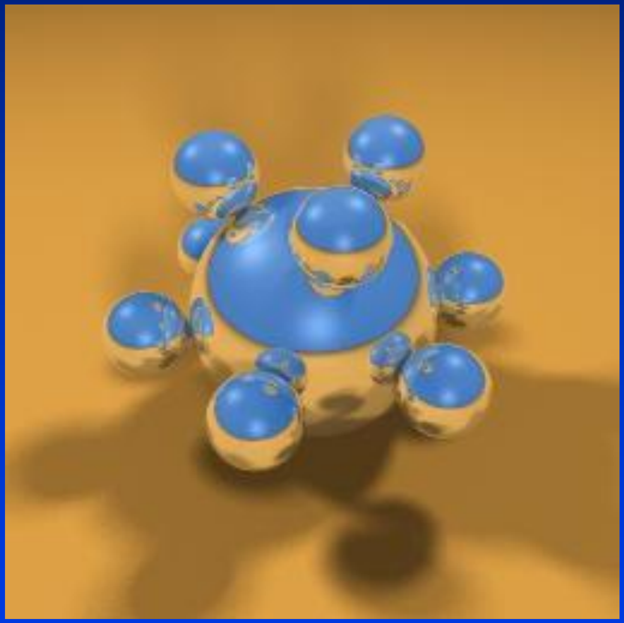
Soft Shadow

- Basic shadow generation was an on/off choice per point
- "Real" shadows do not usually have sharp edges
- Instead of using a point light, use an object with area
- Shoot jittered shadow rays toward the light and count only those that hit it

Soft Shadow Example



Hard shadow



Soft shadow

Radiosity

- Ray tracing models specular reflection and refractive transparency, but still uses an ambient term to account for other lighting effects
- Radiosity is the rate at which energy is emitted or reflected by a surface
- By conserving light energy in a volume, these radiosity effects can be traced

Radiosity – Basic Concept

- Radiosity of a surface: rate at which energy leaves a surface
  - emitted by surface and reflected from other surfaces
- Represent *diffuse* global lighting
- Create a closed energy system where every polygon emits and/or bounces some light at every other polygon
- Calculate how light energy spreads through the system
- Solve a linear system for radiosity of each "surface"
  - Dependent on emissive property of surface
  - Dependent on relation to other surfaces (*form factors*)
- Final output is a polygon mesh with pre-calculated *colors* for each vertex

Radiosity

Radiosity

- Break environment up into a finite number *n* of discrete patches
  - Patches are opaque Lambertian surfaces of finite size
  - Patches emit and reflect light uniformly over their entire surface

Radiosity

- Model light transfer between patches as a system of linear equations
- Solving this system gives the intensity at each patch
- Solve for R, G, B intensities and get color at each patch
- Render patches as colored polygons in OpenGL

Radiosity

- All surfaces are assumed perfectly diffuse
  - What does that mean about property of lighting in scene?
  - Light is reflected equally in all directions
  - Same lighting independent of viewing angle / location
  - Only a subset of the Rendering Equation

Diffuse-diffuse surface lighting effects possible

The "Rendering Equation"

- Jim Kajiya (current head of Microsoft Research) developed this in 1986

$$I(x,x') = g(x,x')\left[\varepsilon(x,x') + \int_S \rho(x,x',x'')I(x',x'')dx''\right]$$

- I(x, x') is the total intensity from point x' to x
- g(x, x') = 0 when x/x' are occluded and $1/d^2$ otherwise (d = distance between x and x')
- ε(x, x') is the intensity emitted by x' to x
- ρ(x, x',x'') is the intensity of light reflected from x'' to x through x'
- S is all points on all surfaces

Radiosity Equation

- Then for each surface *i*:

$$B_i = E_i + \rho_i \sum B_j F_{ji} (A_j / A_i)$$

where

B_i, B_j = radiosity of patch *i*, *j*

A_i, A_j = area of patch *i*, *j*

E_i = energy/area/time emitted by *i*

ρ_i = reflectivity of patch *i*

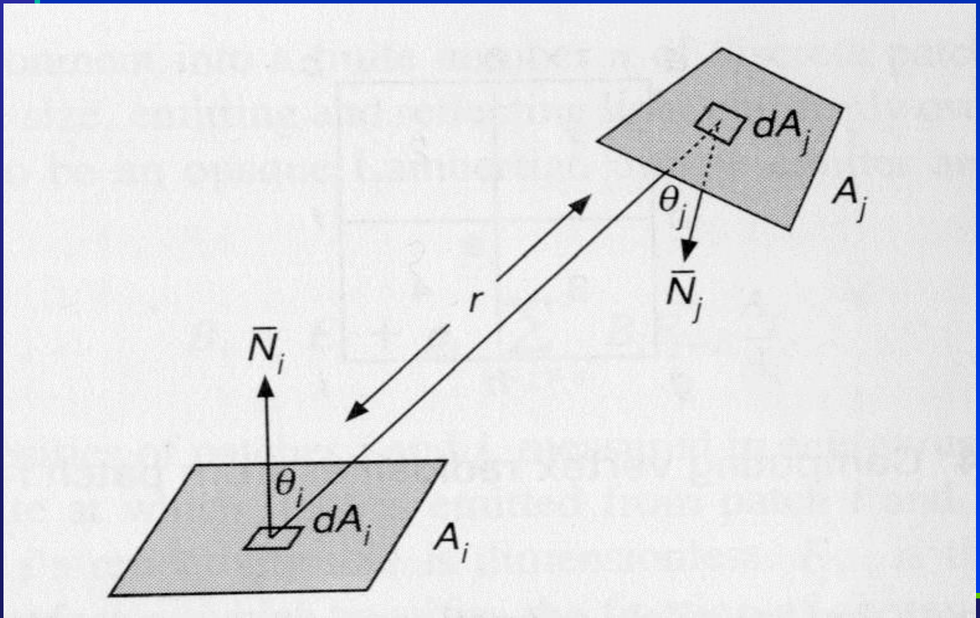
F_{ji} = *Form factor* from *j* to *i*

Form Factors

- *Form factor*: fraction of energy leaving the entirety of patch i that arrives at patch j, accounting for:
  - The shape of both patches
  - The relative orientation of both patches
  - Occlusion by other patches

Form Factors

- Compute n-by-n matrix of form factors to store radiosity relationships between each light patch and every other light patch



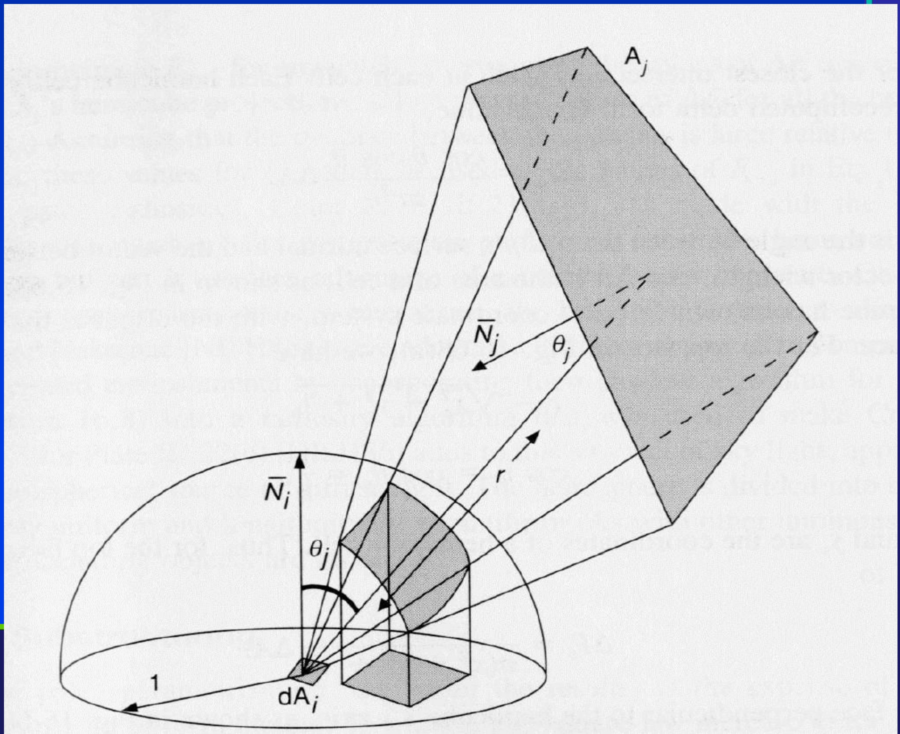
$$dF_{di,dj} = \frac{\cos\theta_i \cos\theta_j}{\pi r^2} H_{ij} dA_j$$

Form Factor – Another Example

- Spherical projections to model form factor
  - Project polygon A_j on unit hemisphere centered at (and tangent to) A_i
    - Contributes $\cos\theta_j / r^2$
  - Project this projection to base of hemisphere
    - Contributes $\cos\theta_i$
  - Divide this area by area of circle base
    - Contributes $\pi(1^2)$

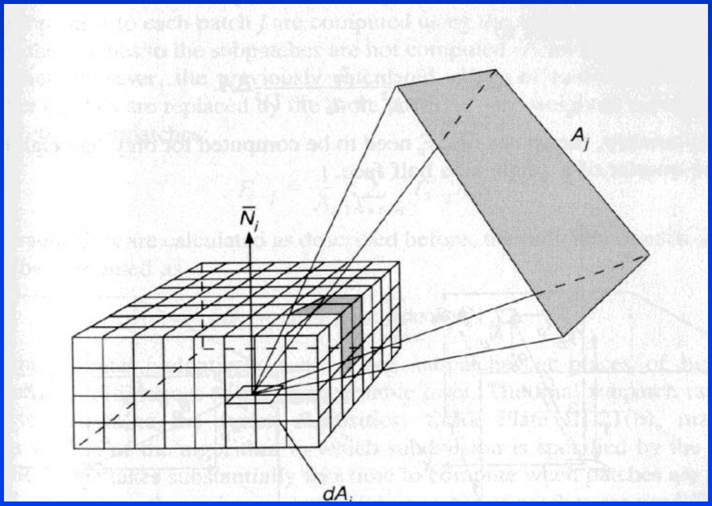
H_{ij} = 1 or 0 depending on occlusion

$$dF_{di,dj} = \frac{\cos\theta_i \cos\theta_j}{\pi r^2} H_{ij} dA_j$$



Form Factor – Another Model

- Hemicube allows faster computations
  - Analytic solution of hemisphere is expensive
  - Use rectangular approximation, Hemicube
  - Cosine terms for top and sides are simplified
  - Dimension of 50 – 200 squares is good



Form Factors Properties

- In diffuse environments, form factors obey a simple reciprocity relationship:

$$A_i F_{ij} = A_j F_{ji}$$

- Which simplifies our equation:

$$B_i = E_i + \rho_i \sum B_j F_{ij}$$

- Rearranging to:

$$B_i - \rho_i \sum B_j F_{ij} = E_i$$

Radiosity Equation

- So…light exchange between all patches becomes a matrix:

$$\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & -\rho_2 F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1-\rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

- *What do the various terms mean?*

Solving Radiosity Equation

Goal

- Find efficient ways to solve the radiosity equation
  - Jacobi Iteration
  - Gauss-Seidel
  - Southwell or Shooting
  - Progressive Radiosity

Radiosity

- Q: *How many form factors must be computed?*
- A: O(n^2)
- Q: *What primarily limits the accuracy of the solution?*
- A: The number of patches

Radiosity

- Now “just” need to solve the matrix!
  - Matrix is “diagonally dominant”
  - Thus Guass-Siedel must converge
- End result: radiosities for all patches
- Solve RGB radiosities separately, color each patch, and render!
- Caveat: actually, color vertices, not patches

Radiosity Equation

$$\begin{bmatrix} 1-\rho_1 F_{1,1} & . & . & . & -\rho_1 F_{1,n} \\ -\rho_2 F_{2,1} & 1-\rho_2 F_{2,2} & . & . & -\rho_2 F_{2,n} \\ . & . & . & . & . \\ . & . & . & . & . \\ -\rho_{n-1} F_{n-1,1} & . & . & . & -\rho_{n-1} F_{n-1,n} \\ -\rho_n F_{n,1} & . & . & . & 1-\rho_n F_{n,n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ . \\ . \\ . \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ . \\ . \\ . \\ E_n \end{bmatrix}$$

- We also need to compute the form factors, F_{ij}
- Problem is the size of matrices (N*N for N elements, N usually > 50000)

Solving for All Patches

- Putting into matrix form
  - $b = e - RFb$
  - $b = [I - RF]^{-1} e$
- Use matrix algebra to solve for B_i's

$$\begin{bmatrix} 1-\rho_1 F_{1-1} & -\rho_1 F_{1-2} & \cdots & -\rho_1 F_{1-n} \\ -\rho_2 F_{2-1} & 1-\rho_2 F_{2-2} & \cdots & -\rho_2 F_{2-n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ -\rho_n F_{n-1} & -\rho_n F_{n-2} & \cdots & 1-\rho_n F_{n-n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \cdot \\ \cdot \\ \cdot \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \cdot \\ \cdot \\ \cdot \\ E_n \end{bmatrix}.$$

Solving for All Patches

- One patch defined by:

$$B_i = \varepsilon_i + \rho_i \sum_{1 \le j \le n} B_j F_{j,i} \frac{A_j}{A_i}$$

- Symmetry: $A_i F_{i,j} = A_j F_{j,I}$

$$B_i = \varepsilon_i + \rho_i \sum_{1 \le j \le n} B_j F_{i,j}$$

- Therefore:

$$B_i - \rho_i \sum_{1 \le j \le n} B_j F_{i,j} = \varepsilon_i$$

Solving for All Patches

- Difficult to perform Gaussian Illumination and solve for b (size of F is large but sparse – why?)

- Instead, iterate: $b^{k+1} = e - RFb^k$

  – Multiplication of sparse matrix is O(n), not O(n_2)

  – Stop when $b^{k+1} = b^k$

Solving for All Patches

- Alternative solution
  - We know:

$$\frac{1}{1-x} = \sum_{i=0}^{\infty} x^i$$

  - Therfore:

$$[I - RF]^{-1} = \sum_{i=0}^{\infty} (RF)^i$$

  - And solution for b is:

$$b = \sum_{i=0}^{\infty} (RF)^i e$$

$$b = e + (RF)e + (RF)^2 e + (RF)^3 e + \cdots$$

Convergence

- Gauss-Seidel known to converge for diagonally dominant matrices

$$
\begin{bmatrix}
1-\rho_1 F_{1,1} & . & . & . & -\rho_1 F_{1,n} \\
-\rho_2 F_{2,1} & 1-\rho_2 F_{2,2} & . & . & -\rho_2 F_{2,n} \\
. & . & . & . & . \\
. & . & . & . & . \\
-\rho_{n-1} F_{n-1,1} & . & . & . & -\rho_{n-1} F_{n-1,n} \\
-\rho_n F_{n,1} & . & . & . & 1-\rho_n F_{n,n}
\end{bmatrix}
\begin{bmatrix}
B_1 \\ B_2 \\ . \\ . \\ . \\ B_n
\end{bmatrix}
=
\begin{bmatrix}
E_1 \\ E_2 \\ . \\ . \\ . \\ E_n
\end{bmatrix}
$$

Solve by Direct Methods?

- Not feasible to use something like Gaussian elimination because of size of matrix
- We don't even want to store the matrix
- Use iterative methods

Radiosity

- Where we go from here:
  - Evaluating form factors
  - *Progressive radiosity*: viewing an approximate solution early
  - *Hierarchical radiosity*: increasing patch resolution on an as-needed basis

Iterative Approach

- Define a residual $r = \mathbf{E} - \mathbf{KB}$
- Iterate, computing **B**, to reduce residual

$$r^{(0)} = \mathbf{E} - \mathbf{KB}^{(0)}$$

- Every iteration, compute new **B** and *r*

$$r^{(k)} = \mathbf{E} - \mathbf{KB}^{(k)}$$

- Initial Condition

$$\mathbf{B}^{(0)} = \mathbf{E}$$

Method 1: Jacobi Iteration

- Update each element $B_i^{(k)}$ to the next iteration using the solution vector $\mathbf{B}^{(k+1)}$ from the previous iteration $\mathbf{B}^{(k)}$
- In other words, compute complete set of **B** and use that for next iteration

Details

- The *i*-th matrix row is $\sum_{j=1}^{n} K_{ij} B_j = E_i$
- Solve for B_i

$$K_{ii} B_i = E_i - \sum_{j \neq i} K_{ij} B_j$$

Details

- Recall that $r^{(k)} = \mathbf{E} - \mathbf{K}\mathbf{B}^{(k)}$
- So $$r^{(k)} = E_i - \sum_{j=1}^{n} K_{ij} B_j^{(k)}$$
- or $$r^{(k)} = E_i - \sum_{j \neq i} K_{ij} B_j^{(k)} - K_{ii} B_i^{(k)}$$
- and $$E_i - \sum_{j \neq i} K_{ij} B_j^{(k)} = r^{(k)} + K_{ii} B_i^{(k)}$$

Substitute

$$E_i - \sum_{j \neq i} K_{ij} B_j^{(k)} = r^{(k)} + K_{ii} B_i^{(k)}$$

into

$$K_{ii} B_i = E_i - \sum_{j \neq i} K_{ij} B_j$$

to get

$$K_{ii} B_i^{(k+1)} = r^{(k)} + K_{ii} B_i^{(k)}$$

or

$$B_i^{(k+1)} = \frac{r^{(k)}}{K_{ii}} + B_i^{(k)}$$

Jacobi Iteration

- If we compute residual *r* each iteration, we can compute updated **B**

$$B_i^{(k+1)} = \frac{r^{(k)}}{K_{ii}} + B_i^{(k)}$$

$$r^{(k)} = E_i - \sum_{j=1}^{n} K_{ij} B_j^{(k)}$$

- Works … but converges slowly

Method 2: Gauss-Seidel

- At each step use the most current values in **B**

$$K_{ii}B_i^{(k+1)} = E_i - \sum_{j=1}^{i-1} K_{ij}B_j^{(k+1)} - \sum_{j=i+1}^{n} K_{ij}B_j^{(k)}$$

- Analogous formulation to get

$$B_i^{(k+1)} = \frac{r^{(k)}}{K_{ii}} + B_i^{(k)}$$

- Now must update residuals at each step

Algorithm

Set all B_i to the E_i values

While (not converged) {

  For (i = 1 to n)

    Compute new B_i

}

A full iteration takes $O(n^2)$ – residual update costs O(n) at each step

Method 3: Gathering

- A physical analogy is to think of a node or element as *gathering* light from all of the other elements to arrive at a new estimate
- Each element *j* contributes some radiosity to the radiosity of element *i* as follows

$$\Delta B_i = \rho_i B_j F_{ij}$$

Gathering variant: Southwell

- Very similar, but instead of proceeding in order from *1* to *n*, choose the row with the *highest residual* and update it….
- …that is, gather to the element which received the <u>least light</u> from what it should

Southwell Algorithm

- For *i*, such that $r_i = Max(\mathbf{r})$, compute

$$B_i^{(k+1)} = E_i - \sum_{j \neq i} \frac{K_{ij} B_j^{(k)}}{K_{ii}}$$

- Note that, now the variable *k* is a **step** and not a **complete** iteration

Complexity

- In order to keep each step *O(n)*, you need to incrementally update the residuals

Computing Residual

- Define the difference in radiosity at each step as

$$\Delta \mathbf{B}^{(p)}$$

- Then

$$\mathbf{B}^{(p+1)} = \mathbf{B}^{(p)} + \Delta \mathbf{B}^{(p)}$$

so the residual can be computed as

$$\mathbf{r}^{(p+1)} = \mathbf{E} - \mathbf{K}(\mathbf{B}^{(p)} + \Delta \mathbf{B}^{(p)}) = \mathbf{r}^{(p)} - \mathbf{K}\Delta \mathbf{B}^{(p)}$$

Only One *B* Changes

- All of the changes in the **B** vector are 0, except for the one that was just updated at step *I*, so

$$r_j^{(p+1)} = r_j^{(p)} - K_{ji}\Delta B_i, \forall j$$

Initial Conditions

- Set $\mathbf{B}^{(0)}$ to all be zero, and $\mathbf{r}^{(0)}$ to be $\mathbf{E}$
- So at the first step, the element being the brightest emitter would have its radiosity set to the value of that emitter and its residual set to 0
- This leads to the interpretation of . . .

Shooting

- The residual can be interpreted as the amount of energy left to be reflected (or emitted)
- At each step, one of the residuals (the one for row *i*) contributes – *shoots* – to all of the other residuals

Progressive Radiosity
(Similar to Southwell)

- Shoot from the element having the most energy
- Compute the form factors *as you shoot*
- Update all of the radiosities
- Display the results every iteration

Initially

For all i {

$B_i = E_i$;

$\Delta B_i = E_i$;

}

```
while (not converged) {
    Select i, such that ΔB_i A_i is greatest;
    Project all other elements onto Hemicube at i to
    compute form factors;
    For every element j {
        ΔRad = ΔB_i * ρ_j F_ji ;
        ΔB_j += ΔRad ;
        B_j += ΔRad ;
    }
    ΔB_i = 0;
    Display image;
}
```

Advantages

- You see progresses
- You don't store a $O(n^2)$ matrix of form factors
- When the process starts out, all of the unshot energy is at lights
- As the process unfolds, the energy is spread around and the residuals become more even

Ambient Term

- An estimate of the average form factors can be made from their areas

$$F_{*j} \approx \frac{A_j}{\sum_{j=1}^{n} A_j}$$

- We can also compute the area-weighted average of reflectivities

$$\overline{\rho} = \frac{\sum \rho_i A_i}{\sum A_i}$$

Ambient Term

- Just to make the images look better (less dark) at the beginning, Cohen, et. al. use an ambient term
- It's related to the reflected illumination not yet accounted for (or in other words the energy yet unshot)

Ambient Estimate

- Ambient term is total of the area-weighted unshot energy times the total reflectivity

$$B_{ambient} = R_{total} \sum_{j=1}^{n} (\Delta B_j F_{*j})$$

- Each element displays its own fraction

$$B_i^{display} = B_i + \rho_i B_{ambient}$$

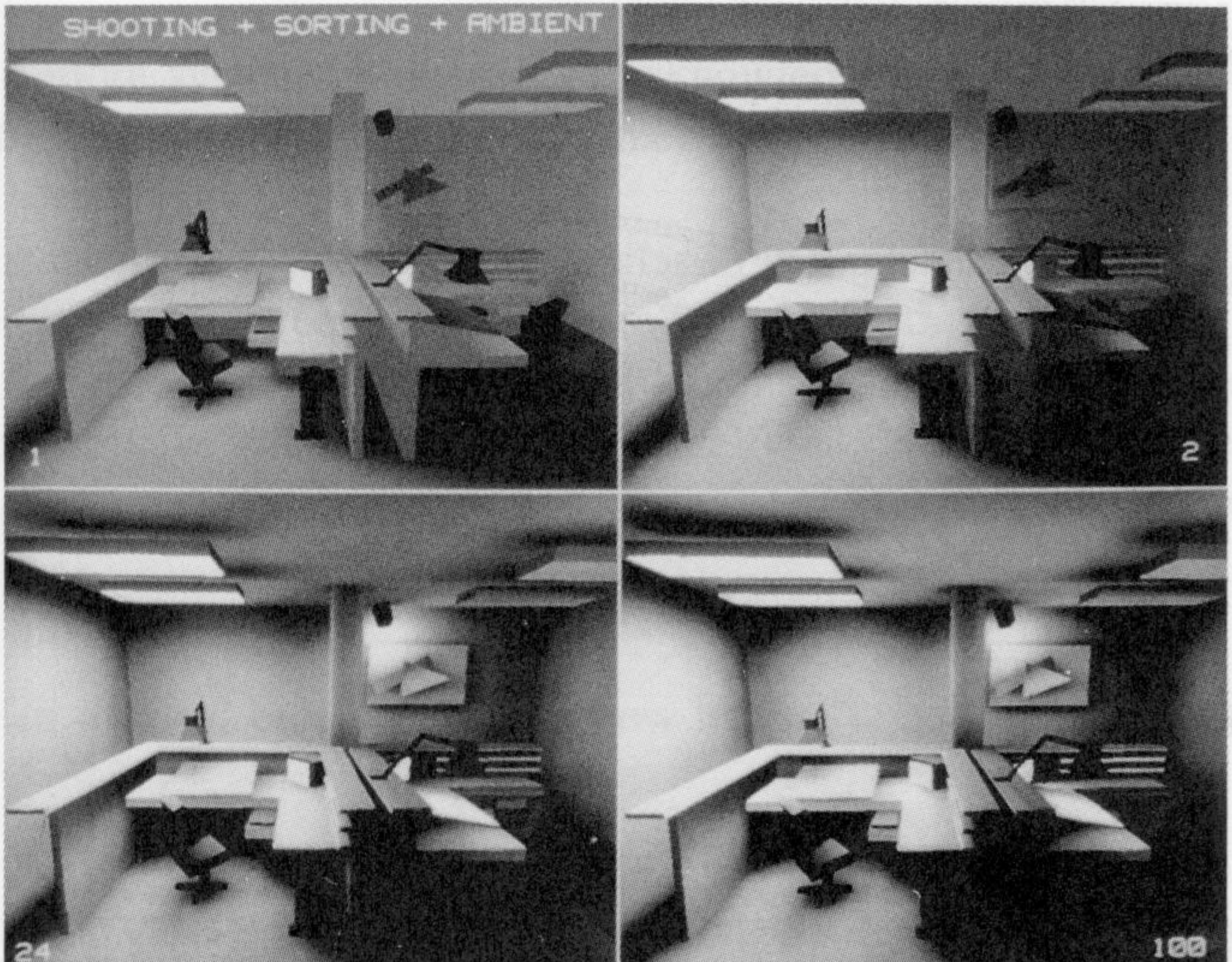
Reflection

- The energy will be reflected over and over, so the total reflection can be expressed as

$$R_{total} = 1 + \bar{\rho} + \bar{\rho}^2 + \bar{\rho}^3 + \ldots = \frac{1}{1 - \bar{\rho}}$$

- 30,000 patches divided into 50,000 elements.
- Solution run for only 2000 patches
- View-dependent post-process, computing radiosity at visible vertices, 190 hours

Ex



Displayed Image after 1, 2, 24, and 100 Steps

Magritte Studio Image

Radiosity - Cons

- Form factors need to be re-computed if *anything* moves
- Large computational and storage costs
- Non-diffuse light not represented
  - Mirrors and shiny objects hard to include
- Lighting effects tend to be "blurry", not sharp without good *subdivision*
- Not applicable to procedurally defined surfaces

Radiosity - Pros

- Viewpoint independence means fast real-time display after initial calculation
- Inter-object interaction possible
  - Soft shadows
  - Indirect lighting
  - Color bleeding
- Accurate simulation of energy transfer

View-dependent vs View-independent

- Ray-tracing models specular reflection well, but diffuse reflection is approximated
- Radiosity models diffuse reflection accurately, but specular reflection is ignored
- Advanced algorithms combine the two

Radiosity

- Radiosity is expensive to compute
- Some parts of illuminated world can change
  - Emitted light
  - Viewpoint
- Other things cannot
  - Light angles
  - Object positions and occlusions
  - Computing form factors is expensive
- Specular reflection information is not modeled

Summary

- Now we know
  - How to formulate the radiosity problem
  - How to solve equations
  - How to approximate form factors

References

- Cohen and Wallace, Radiosity and Realistic Image Synthesis, Chapter 5.