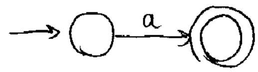
A SIMILAR EXAMPLE TO PROBLEM 1:

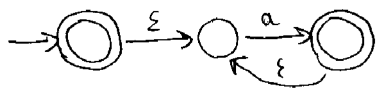
$$L = a^* \cup (a\,b)^*$$

FOLLOWING THE STEPS OF LEMMA 1.55 WE GET

a :

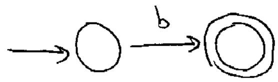


a^* :

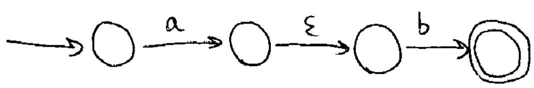


(SEE PAGE 62 FOR THIS)

b :

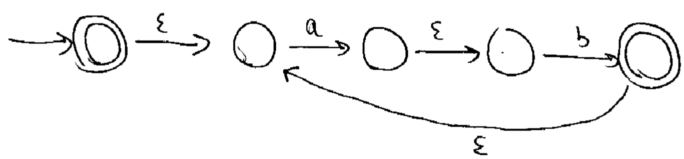


$a\,b$:

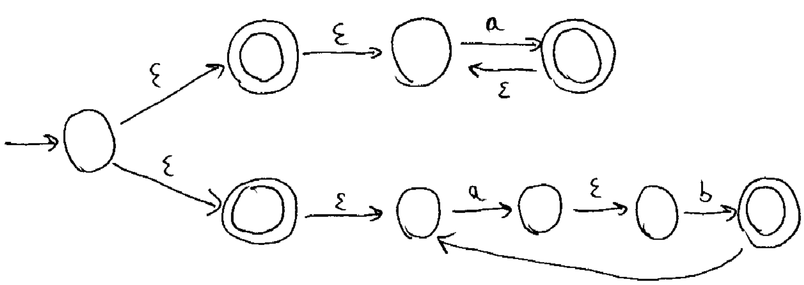


(SEE PAGE 61)

$(a\,b)^*$:



$a^* \cup (a\,b)^*$:

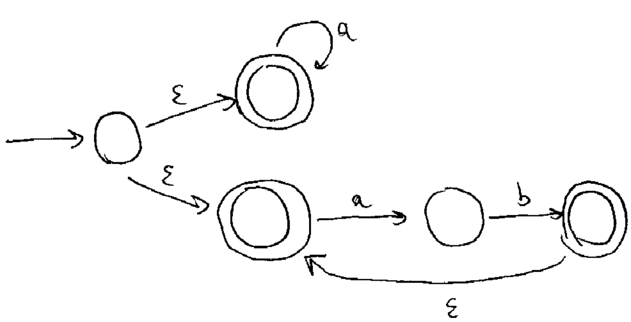


(SEE PAGE 59)

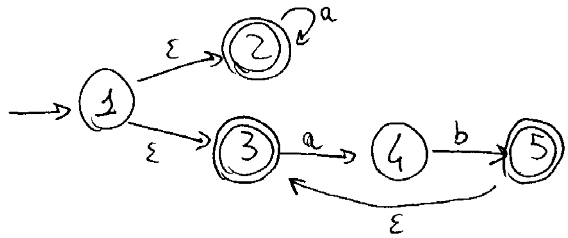
THIS IS OBTAINED FOLLOWING EVERY SINGLE STEP PRECISELY

BUT IF YOU SEE A CLEAR SEMPLIFICATION, YOU CAN SIMPLIFY TO REDUCE THE NUMBER OF STATES AND MAKE THE PROBLEM EASIER FOR THE NEXT STEP

FOR EXAMPLE



THE 2^{ND} PART REQUIRES TO TRANSFORM THIS DFA

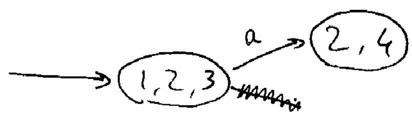


INTO A DFA

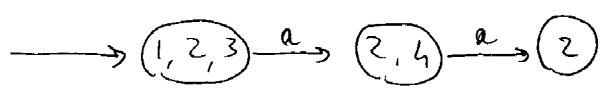
WE START FROM 1, THE START STATE

A "ε" ALLOWS TO MOVE DIRECTLY TO THE DESTINATION STATES. SO BEING IN 1 IS LIKE BEING IN 2 AND 3 AT THE SAME TIME. FOR THIS REASON WE GET A COMBINED STATE

TO UNDERSTAND HOW IT WORKS, PUT YOUR FINGERS ON 1, 2 AND 3. WHERE CAN YOU GO WITH AN "a"? YOU END UP IN 2 AND IN 4. THIS MEANS THAT THERE IS A STATE (2, 4):

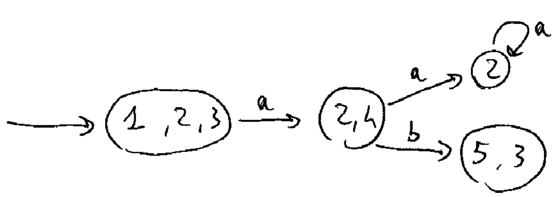


NOW WE ARE DONE WITH (1,2,3). WHERE DO WE GO FROM (2,4) READING AN "a"? ONLY TO 2, SO THIS GIVES A NEW STATE WITH 2 ONLY

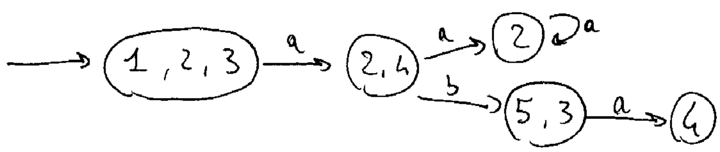


FROM 2, WITH a WE LOOP AROUND.

FROM (2,4) WITH b WE ARRIVE TO 5. BUT 5 HAS AN OUTGOING ε, WHICH MEANS THAT BEING IN 5 IS LIKE BEING IN 3 TOO

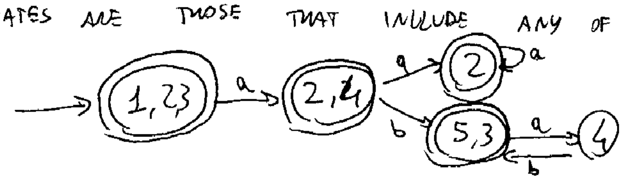


NOW FROM 3 WE CAN GO TO 4 WITH AN a, BUT THIS IS NOT (2,4), SO



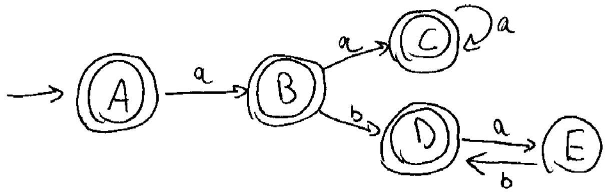
FINALLY FROM 4 WE CAN GO BACK TO 5,3 WITH A b.

THE FINAL STATES ARE THOSE THAT INCLUDE ANY OF THE ORIGINAL ONES:



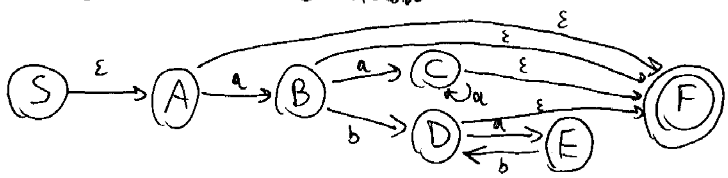
IT IS VERY IMPORTANT TO DOUBLE CHECK THE RESULT, BECAUSE IT IS VERY EASY TO FORGET SOMETHING...

THE FINAL PART CONSISTS IN CONVERTING BACK THE DFA

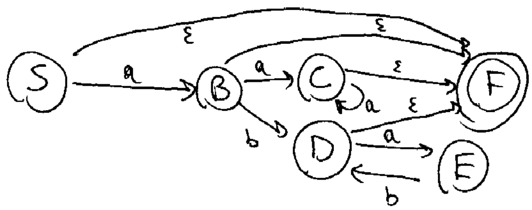


INTO A REGULAR EXPRESSION.

WE NEED TO ADD A START STATE AND A FINAL STATE WHICH ~~HAVE~~ SHOULD BE CONNECTED TO ALL FINAL STATES WITH AN ε- ARROW

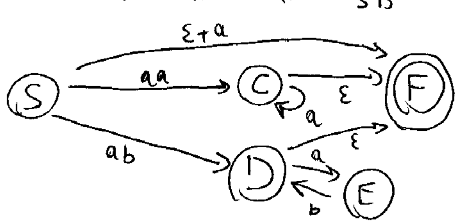


WE ELIMINATE ONE STATE AT A TIME STARTING FROM A (THE ORDER DOES NOT MATTER)
IF A IS NOT THERE, WE STILL NEED TO REACH THE OTHER STATES IN THE SAME WAY AS BEFORE, SO "εa" WOULD BRING TO B AND "εε" TO F

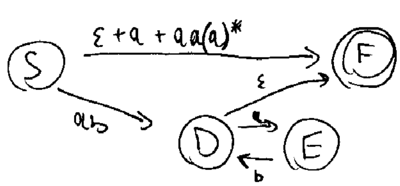


NOW WE REMOVE B. LET'S LOOK AT ALL THE DESTINATIONS OF B:

BF : THIS MEANS THAT SF SHOULD INCLUDE "aε"
BC : " " " SC " " "aa"
SD : " " " SD " " "ab"

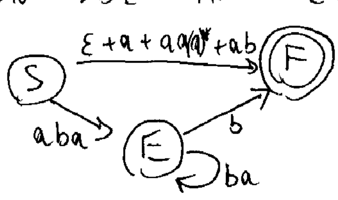


NOW WE REMOVE C : THE LOOP TRANSLATES INTO a^* SO WE GET



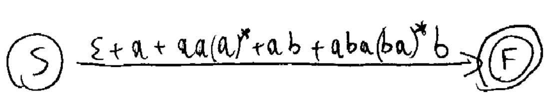
MAYBE REMOVING E WOULD MAKE THE LIFE EASIER, BUT JUST TO MAKE THIS EXAMPLE MORE INTERESTING WE FOLLOW THE DIFFICULT PATH...

WHEN WE REMOVE D, WE SHOULD PAY ATTENTION ON ~~HOW~~ THE THE PATH S→D→F AND ALSO ON SDE AND EDE :

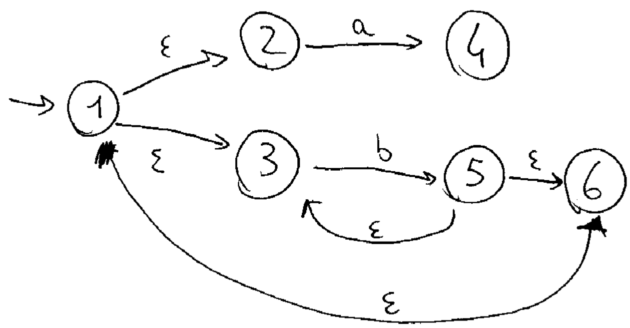


THIS WAS THE MOST CRITICAL STEP, PLEASE MAKE SURE YOU UNDERSTAND ALL DETAILS

FINALLY :



ONE MORE EXAMPLE FROM NFA TO DFA



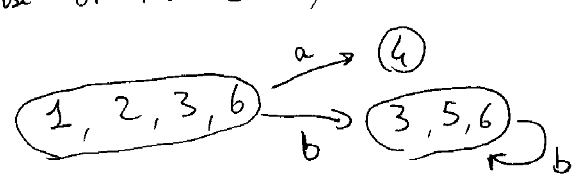
AT THE BEGINNING WE ARE IN 1, BUT BECAUSE OF ε WE ARE ALSO IN 2, 3 AND 6, SO THE FIRST STATE IS

1, 2, 3, 6

FROM 2 WITH AN a WE GO TO 4, SO

$1,2,3,6 \xrightarrow{a} 4$

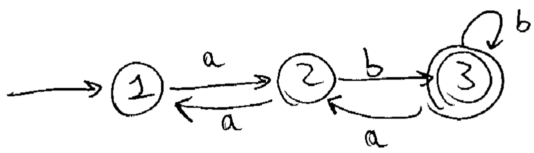
AND FROM 3, WE GO TO 5 WITH A b, BUT 5 IS ALSO 3 AND 6 BECAUSE OF THE ε, SO



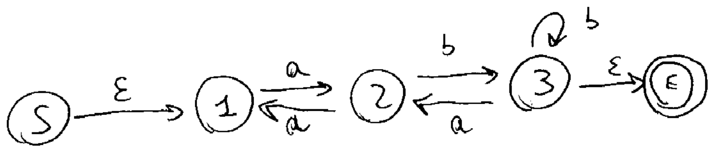
THIS SELF LOOP IS BECAUSE FROM 3 WE CAN AGAIN GO TO 356

NOTHING ELSE TO DO

ONE MORE EXAMPLE FROM DFA TO REGULAR EXPRESSION

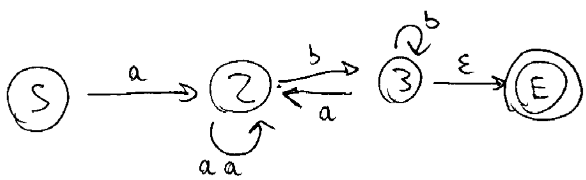


FIRST WE ADD A START AND END STATE

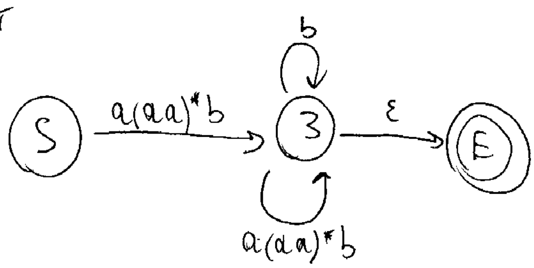


NOW WE ELIMINATE ONE STATE AT A TIME

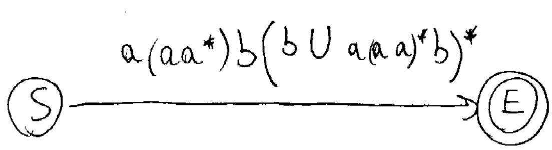
IF WE ELIMINATE STATE 1, WE SHOULD STILL BE ABLE TO REACH 2 WITH AN a AND COME BACK TO 2 WITH TWO a's ("aa")



WHAT HAPPENS IF WE ELIMINATE 2?
WE SHOULD STILL BE ABLE TO REACH 3 WITH $a(aa)^*b$
BUT WE ALSO NEED TO BE ABLE TO LOOP AROUND 3 $a(aa)^*b$
(BECAUSE FROM 3 WE CAN GO TO 2 WITH AN a, LOOP AROUND WITH aa AND COME BACK WITH b)
SO WE GET



FINALLY WE ELIMINATE 3. THERE ARE TWO WAYS TO LOOP AROUND 3, SO THAT REPRESENTS A UNION OPERATION



THEN THE ANSWER IS $a(aa^*)b(b \cup a(aa)^*b)^*$