CSE371 Q1 SOLUTIONS SPRING 2024
(2pt)

ONE PROBLEM (2pts)

Part 1 Write the following natural language statement:

From the fact that there is a blue bird we deduce that: it is not necessary that all natural numbers are even OR, if it is possible that it is not true that all natural numbers are even, then it is not true that there is a blue bird.

As a formula $A \in \mathcal{F}$ of a propositional language with the set $\{\neg, \cap, \cup, \Rightarrow\}$ of propositional connectives'

Use a, b, c... for Propositional variables and you must write which variables denote which sentences)

Solution Propositional Variables are: a, b, c
a denotes statement: *there is a blue bird* ,
b denotes statement: *it is necessary that all natural numbers are even*,
c denotes statement: *possible that it is not true that all natural numbers are even*

Formula $A \in \mathcal{F}$ is:

$$(a \Rightarrow (\neg b \cup (c \Rightarrow \neg a)))$$

PART 2 Here is a mathematical statement **S**:

For each integer $m \in Z$ the following holds: If $m > 5$, then there is a natural number $n \in N$, such that $m + n > 5$

Re-write **S** as a symbolic mathematical statement **SM** that only uses mathematical and logical symbols.

Solution **S** becomes a symbolic mathematical statement

$$\textbf{SM}: \quad \forall_{m \in Z}(m > 5 \Rightarrow \exists_{n \in N}\ m + n > 5)$$

Translate the mathematical statement **SM** into **formula** with **restricted quantifiers** of a to a corresponding predicate language $\mathcal{L}$. Explain your choice of symbols.

Solution We write Z(x) for $x \in Z$, N(y) for $y \in N$, a constant c for the number 5. We use $G \in \mathbf{P}$ to denote the relation $>$, we use $f \in \mathbf{F}$ to denote the function $+$.

The statement $m > 5$ becomes an **atomic formula** G(x, c). The statement $m + n > 5$ becomes an **atomic formula** G(f(x,y), c).

The symbolic mathematical statement **SM** becomes a **restricted quantifiers** formula

$$\forall_{Z(x)}(G(x, c) \Rightarrow \exists_{N(y)} G(f(x, y), c))$$

Translate your **restricted domain** quantifiers formula into a **correct formula** A of the predicate language $\mathcal{L}$

Solution We apply now the **transformation rules** and get a corresponding formula $A \in \mathcal{F}$:

$$\forall x(Z(x) \Rightarrow (G(x, c) \Rightarrow \exists y(N(y) \cap G(f(x, y), c))))$$