CSE/MAT371 Extra Q2 SOLUTIONS SPRING 2024
(2pt extra credit)

ONE PROBLEM (2pts)

Part 1 (1pts) Write the following natural language statement:
From the fact that there is a bird that does not fly and $4 + 4 = 4$*, we deduce the following: it is not possible that all birds fly OR it is not necessary that* $4 + 4 = 4$.

in the **TWO WAYS**:

WAY 1 (0.5pts) As a formula $A_1 \in \mathcal{F}_1$ of a language $\mathcal{L}_{\{\neg,\ \Box,\ \Diamond,\ \cap,\ \cup,\ \Rightarrow\}}$

SOLUTION Use Propositional Variables a, b, c where
a denotes statement: *there is a bird that does not fly*
b denotes statement: $4 + 4 = 4$ c denotes statement: *all birds fly*

The formula $A_1 \in \mathcal{F}_1$ is:

$$((a \cap b) \Rightarrow (\neg\Diamond c \cup \neg\Box b))$$

WAY 2 (0.5pts) As a formula $A_2 \in \mathcal{F}_2$ of a PREDICATE LANGUAGE language $\mathcal{L}(\mathbf{P},\ \mathbf{F},\ \mathbf{V})$ with the set $\{\neg,\ \Box,\ \Diamond,\ \cap,\ \cup,\ \Rightarrow\}$ of propositional connectives.

Use the following Predicates, Functions and Constants

$B(x)$ for x is a bird, $F(x)$ for x can fly, $E(x, y)$ for $x = y$, $f(x, y)$ for $+$, and c for 4.

(0.2pts) Restricted domain formula is:

$$((\exists_{B(X)}\neg F(x) \cap E(f(c, c), c)) \Rightarrow (\neg\Diamond\forall_{B(X)}F(x) \cup \neg\Box E(f(c, c), c)))$$

(0.3pts) Formula $A_2 \in \mathcal{F}_2$ is:

$$((\exists x(B(X) \cap \neg F(x)) \cap E(f(c, c), c)) \Rightarrow (\neg\Diamond\forall x(B(X) \Rightarrow F(x)) \cup \neg\Box E(f(c, c), c)))$$

Part 2 (1pts)

(0.5pts) **Circle** formulas that **are propositional tautologies**

(0.5pts) **Circle** formulas that **are propositional tautologies**

$$\mathcal{S}_1 = \{\ ((\neg c \cap c) \Rightarrow (\neg b \Rightarrow (d \cap e))),\quad ((a \Rightarrow b) \cup \neg(a \Rightarrow b)),\quad ((a \cap \neg b) \Rightarrow ((a \cap \neg b) \Rightarrow (\neg d \cup e))),\quad (\neg a \Rightarrow (\neg a \cup b))\ \}$$

Solution $\not\models ((a \cap \neg b) \Rightarrow ((a \cap \neg b) \Rightarrow (\neg d \cup e)))$, all other formulas are tautologies

(0.5pts) **Circle** formulas that **are predicate tautologies**

$$\mathcal{S}_2 = \{\ (\exists x\, A(x) \Rightarrow \neg\forall x \neg\, A(x)),\quad (\forall x\, (P(x, y) \cap Q(y)) \Rightarrow \neg\exists x\, \neg(P(x, y) \cap Q(y))),$$

$$((\exists x A(x) \cap \exists x B(x)) \Rightarrow \exists x\, (A(x) \cap B(x))),\quad (\forall x(A(x) \Rightarrow B)\ \Rightarrow\ (\exists x A(x) \Rightarrow B))\ \}$$

Solution $\not\models ((\exists x A(x) \cap \exists x B(x)) \Rightarrow \exists x\, (A(x) \cap B(x)))$, all other formulas are tautologies