CSE/MAT371 Extra Q4 SOLUTIONS SPRING 2024
(3pts extra credit)

ONE PROBLEM (3pts)

Part 1 (1.5pts)

1. (0.5pts) Given $\mathcal{L} = \mathcal{L}_{\{\neg,\Rightarrow,\cup,\cap\}}$ and **classical semantics**. We **define:** A set $\mathcal{G} \subseteq \mathcal{F}$ is **consistent** if and only if there is a truth assignment v such that $v \models \mathcal{G}$

PROVE that the set $\mathcal{G} = \{(a \Rightarrow (a \cup b)),\ (a \cup b),\ \neg b,\ (c \Rightarrow b)\}$ is **consistent**. Use **shorthand** notation.

Solution: We find a restricted model for $\mathcal{G}$ as follows

First observe that the formula $((a \Rightarrow a \cup b))$, is a tautology, hence any v is its model. So we have only to see whether two other formulas have a common model. It means we check if it is possible to find v, such that $v^*(\neg b) = T$, $v^*((a \cup b)) = T$, and $v^*((c \Rightarrow b)) = T$.

We have that $\neg b = T$ if and only if $b = F$.

We evaluate $(a \cup b) = (a \cup F) = T$ if and only if $a = T$.

Consequently, $(c \Rightarrow b) = (c \Rightarrow F) = T$ if and only if $c = F$.

Hence, any v, such that $a = T,\ b = T,$ and $c = F$ is a model for $\mathcal{G}$.

2. (0.5pts) How many restricted MODELS does $\mathcal{G}$ have?

We proved that $a = T,\ b = T,$ and $c = F$ is the **only restricted model** for $\mathcal{G}$.

3. (0.5pts) We **define:** a formula $A \in \mathcal{F}$ is called **independent** from a set $\mathcal{G} \subseteq \mathcal{F}$ if and only if when there are truth assignments $v_1,\ v_2$ such that $v_1 \models \mathcal{G} \cup \{A\}$ and $v_2 \models \mathcal{G} \cup \{\neg A\}$.

PROVE the a formula $A = (d \cup \neg a)$ is **independent** of $\mathcal{G}$ defined in **1.** Use **shorthand** notation.

Solution

We proved that $a = T,\ b = T,$ and $c = F$ is the **only restricted model** for $\mathcal{G}$

Any v_1 such that $a = T,\ b = T, c = F$, and $d = T$ is a MODEL for $\mathcal{G} \cup \{A\}$

Any v_1 such that $a = T,\ b = T,\ c = F$, and $d = F$ is a MODEL for $\mathcal{G} \cup \{\neg A\}$ as

$\neg A = \neg(d \cup \neg a) = \neg(F \cup F) = T$

ONE PROBLEM PART 2 (1.5pts) Use **shorthand notation**.

1. (0.5pts) Given a formula $A = ((a \cap \neg c) \Rightarrow (c \cup b))$ of a language $\mathcal{L}_{\{\neg,\cap,\cup,\Rightarrow\}}$.

Transform A to a formula B of a language $\mathcal{L}_{\{\neg,\Rightarrow\}}$, such that $A \equiv B$.

Solution

$$((a \cap \neg c) \Rightarrow (c \cup b)) \equiv (\neg(a \Rightarrow \neg\neg c) \Rightarrow (\neg c \Rightarrow b))$$

(0.5pts) **List** all proper logical equivelences defining $\cup,\ \cap$ connectives in terms of $\neg, \Rightarrow$

Equivalences used:

$$(A \cap B) \equiv \neg(A \Rightarrow \neg B) \quad \text{and} \quad (A \cup B) \equiv (\neg A \Rightarrow B)$$

Plus Plus Substitution Theorem.

2. (0.5pts) Given a formula $A = ((a \cap \neg c) \Rightarrow (c \cup b))$ of a language $\mathcal{L}_{\{\neg,\cap,\cup,\Rightarrow\}}$.

Transform it to a formula B of a language $\mathcal{L}_{\{\neg,\cap,\cup\}}$, such that $A \equiv B$.

Solution

$$((a \cap \neg c) \Rightarrow (c \cup b)) \equiv (\neg(a \cap \neg c) \cup (c \cup b)) \quad \text{or} \quad \neg((a \cap \neg c) \cap \neg(c \cup b))$$

List all proper logical equivalences defining $\Rightarrow$ in terms of $\neg, \cup$, $\neg, \cap$, respectively.

$$(A \Rightarrow B) \equiv (\neg A \cup B) \quad \text{or} \quad (A \Rightarrow B) \equiv \neg(A \cap \neg B)$$