CSE371 PRACTICE Q1

SOLVE ALL PROBLEMS as PRACTICE and only AFTER look at the SOLUTIONS!!

Write your solutions CAREFULLY and COMPARE with Solutions posted. Some of the problems may appear on real Q1.

QUESTION 1 Give a definition and an example of a default reasoning.

QUESTION 2

1. (4pts) Write the following natural language statement:

From the fact that it is not necessary that an elephant is not a bird we deduce that:
it is not possible that an elephant is a bird or, if it is possible that an elephant is a bird, then it is not necessary that a bird flies.

as a formula

$A_1 \in \mathcal{F}_1$ of a language $\mathcal{L}_{\{\neg, \mathbf{C}, \mathbf{I}, \cap, \cup, \Rightarrow\}}$,

$A_2 \in \mathcal{F}_2$ of a language $\mathcal{L}_{\{\neg, \cap, \cup, \Rightarrow\}}$.

2. (2pts) Main connective of the formula A_1 is: , main connective of the formula A_2 is:

3. Degree of the formula A_1 is: , degree of the formula A_2 is:

4. All proper, non-atomic sub-formulas of A_1 are:

5. All non-atomic sub-formulas of A_2 are:

6. Find a restricted model and a restricted counter-model of A_2. Use short-hand notation. Show work.

A Restricted Model:

Evaluation:

A Restricted Counter-Model:

Evaluation:

7. There are more then 3 possible restricted counter-models of A_2. Justify.

8. There are more then 2 possible restricted models of A_2. Justify your answer.

9. List 3 models and 2 counter-models for A_2 by extending the restricted model and the counter-model you have found in **6.** to the set VAR of all variables.

10. There are possible models for A_2.
There are possible counter-models for A_2.

QUESTION 3 Show that

$$\models (\neg((a \cap \neg b) \Rightarrow ((c \Rightarrow (\neg f \cup d)) \cup e)) \Rightarrow ((a \cap \neg b) \cap (\neg(c \Rightarrow (\neg f \cup d)) \cap \neg e))).$$

REMINDER: We define **H** semantics operations $\cup$ and $\cap$ as follows

$$a \cup b = max\{a, b\}, \quad a \cap b = min\{a, b\}.$$

The Truth Tables for Implication and Negation are:

H-Implication

$\Rightarrow$	F	$\bot$	T
F	T	T	T
$\bot$	F	T	T
T	F	$\bot$	T

H Negation

$\neg$	F	$\bot$	T
	T	F	F

QUESTION 4 **We know** that

$$v : VAR \longrightarrow \{F, \bot, T\}$$

is such that

$$v^*((a \cap b) \Rightarrow (a \Rightarrow c)) = \bot$$

under **H** semantics. **evaluate** $v^*(((b \Rightarrow a) \Rightarrow (a \Rightarrow \neg c)) \cup (a \Rightarrow b))$.

QUESTION 5

We define a 4 valued $\mathbf{Ł}_4$ logic semantics as follows. The language is $\mathcal{L} = \mathcal{L}_{\{\neg, \Rightarrow, \cup, \cap\}}$. The logical connectives $\neg, \Rightarrow, \cup, \cap$ of $\mathbf{Ł}_4$ are operations in the set $\{F, \bot_1, \bot_2, T\}$, where $\{F < \bot_1 < \bot_2 < T\}$, defined as follows.

Negation $\neg$ is a function $\neg : \{F, \bot_1, \bot_2, T\} \longrightarrow \{F, \bot_1, \bot_2, T\}$, such that

$$\neg\bot_1 = \bot_1, \quad \neg\bot_2 = \bot_2, \quad \neg F = T, \quad \neg T = F.$$

Conjunction $\cap$ is a function $\cap: \{F, \bot_1, \bot_2, T\} \times \{F, \bot_1, \bot_2, T\} \longrightarrow \{F, \bot_1, \bot_2, , T\}$, such that for any $a, b \in \{F, \bot_1, \bot_2, T\}$, $a \cap b = min\{a, b\}$.

Disjunction $\cup$ is a function $\cup: \{F, \bot_1, \bot_2, T\} \times \{F, \bot_1, \bot_2, T\} \longrightarrow \{F, \bot_1, \bot_2, T\}$, such that for any $a, b \in \{F, \bot_1, \bot_2, T\}$, $a \cup b = max\{a, b\}$.

Implication $\Rightarrow$ is a function $\Rightarrow$: $\{F, \bot_1, \bot_2, T\} \times \{F, \bot_1, \bot_2, T\} \longrightarrow \{F, \bot_1, \bot_2, T\}$, such that for any $a, b \in \{F, \bot_1, \bot_2, T\}$,

$$a \Rightarrow b = \begin{cases} \neg a \cup b & \text{if } a > b \\ T & \text{otherwise} \end{cases}$$

Part 1 Write all Truth Tables for **Ł**$_4$

Solution :

Part 2 Verify whether

$$\models_{Ł_4} ((a \Rightarrow b) \Rightarrow (\neg a \cup b))$$

Solution :