

CSE371 Q2 PRACTICE Fall 2009

NAME

ID:

Math/CS

Solve all problems as an exercise.

Reminder: We define **H** semantics operations \cup and \cap as follows

$$a \cup b = \max\{a, b\}, \quad a \cap b = \min\{a, b\}.$$

The **Truth Tables** for Implication and Negation are:

H-Implication

\Rightarrow	F	\perp	T
F	T	T	T
\perp	F	T	T
T	F	\perp	T

H Negation

\neg	F	\perp	T
	T	F	F

QUESTION 1 We know that

$$v : VAR \longrightarrow \{F, \perp, T\}$$

is such that

$$v^*((a \cap b) \Rightarrow (a \Rightarrow c)) = \perp$$

under **H** semantics. **evaluate** $v^*((((b \Rightarrow a) \Rightarrow (a \Rightarrow \neg c)) \cup (a \Rightarrow b)))$.

QUESTION 2

We **define** a 4 valued \mathbf{L}_4 logic semantics as follows. The language is $\mathcal{L} = \mathcal{L}_{\{\neg, \Rightarrow, \cup, \cap\}}$. The logical connectives $\neg, \Rightarrow, \cup, \cap$ of \mathbf{L}_4 are operations in the set $\{F, \perp_1, \perp_2, T\}$, where $\{F < \perp_1 < \perp_2 < T\}$, defined as follows.

Negation \neg is a function $\neg: \{F, \perp_1, \perp_2, T\} \longrightarrow \{F, \perp_1, \perp_2, T\}$, such that

$$\neg \perp_1 = \perp_1, \quad \neg \perp_2 = \perp_2, \quad \neg F = T, \quad \neg T = F.$$

Conjunction \cap is a function $\cap: \{F, \perp_1, \perp_2, T\} \times \{F, \perp_1, \perp_2, T\} \longrightarrow \{F, \perp_1, \perp_2, T\}$, such that for any $a, b \in \{F, \perp_1, \perp_2, T\}$, $a \cap b = \min\{a, b\}$.

Disjunction \cup is a function $\cup: \{F, \perp_1, \perp_2, T\} \times \{F, \perp_1, \perp_2, T\} \longrightarrow \{F, \perp_1, \perp_2, T\}$, such that for any $a, b \in \{F, \perp_1, \perp_2, T\}$, $a \cup b = \max\{a, b\}$.

Implication \Rightarrow is a function $\Rightarrow: \{F, \perp_1, \perp_2, T\} \times \{F, \perp_1, \perp_2, T\} \longrightarrow \{F, \perp_1, \perp_2, T\}$, such that for any $a, b \in \{F, \perp_1, \perp_2, T\}$,

$$a \Rightarrow b = \begin{cases} \neg a \cup b & \text{if } a > b \\ T & \text{otherwise} \end{cases}$$

Part 1 Write all Truth Tables for \mathbf{L}_4

Solution :

Part 2 Verify whether

$$\models_{\mathbf{L}_4} ((a \Rightarrow b) \Rightarrow (\neg a \cup b))$$

Solution :

QUESTION 3 Prove using proper logical equivalences (list them at each step) that

1. $\neg(A \Leftrightarrow B) \equiv ((A \cap \neg B) \cup (\neg A \cap B)),$

Solution :

2. $((B \cap \neg C) \Rightarrow (\neg A \cup B)) \equiv ((B \Rightarrow C) \cup (A \Rightarrow B)).$

Solution :

QUESTION 4 We define an EQUIVALENCE of LANGUAGES as follows:

Given two languages:

$\mathcal{L}_1 = \mathcal{L}_{CON_1}$ and $\mathcal{L}_2 = \mathcal{L}_{CON_2}$, for $CON_1 \neq CON_2$.

We say that they are **logically equivalent**, i.e.

$$\mathcal{L}_1 \equiv \mathcal{L}_2$$

if and only if the following conditions **C1**, **C2** hold.

C1: For every formula A of \mathcal{L}_1 , there is a formula B of \mathcal{L}_2 , such that

$$A \equiv B,$$

C2: For every formula C of \mathcal{L}_2 , there is a formula D of \mathcal{L}_1 , such that

$$C \equiv D.$$

Prove that $\mathcal{L}_{\{\neg, \cap\}} \equiv \mathcal{L}_{\{\neg, \Rightarrow\}}$.

QUESTION 5 Given a proof system:

$$S = (\mathcal{L}_{\{\neg, \Rightarrow\}}, \mathcal{E} = \mathcal{F} \quad AX = \{(A \Rightarrow A), (A \Rightarrow (\neg A \Rightarrow B))\}, \quad (r) \frac{(A \Rightarrow B)}{(B \Rightarrow (A \Rightarrow B))}).$$

Definition: System S is sound if and only if

- (i) Axioms are tautologies and
- (ii) rules of inference are sound, i.e lead from all true premisses to a true conclusion.

1. Prove that S is *sound* under classical semantics.

2. Prove that S is *not sound* under \mathbf{K} semantics defined as follows.

The language is the same in case of classical logic.

Connectives \neg, \cup, \cap of \mathbf{K} are defined as in \mathbf{L} logic, i.e. for any $a, b \in \{F, \perp, T\}$,

$$\neg \perp = \perp, \quad \neg F = T, \quad \neg T = F,$$

$$a \cup b = \max\{a, b\},$$

$$a \cap b = \min\{a, b\}.$$

Implication in Kleene's logic is defined as follows.

For any $a, b \in \{F, \perp, T\}$,

$$a \Rightarrow b = \neg a \cup b.$$

3. Write a formal proof in S with 2 applications of the rule (r).

QUESTION 6 Prove, by constructing a formal proof that

$$\vdash_S ((\neg A \Rightarrow B) \Rightarrow (A \Rightarrow (\neg A \Rightarrow B))).$$