

CSE371 Q3 PRACTICE Fall 2010

QUESTION 1

H is the following proof system:

$$H = (\mathcal{L}_{\{\Rightarrow, \neg\}}, \mathcal{F}, AX = \{A1, A2, A3, A4\}, MP)$$

A1 $(A \Rightarrow (B \Rightarrow A)),$

A2 $((A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C))),$

A3 $((\neg B \Rightarrow \neg A) \Rightarrow ((\neg B \Rightarrow A) \Rightarrow B))$

A4 $((A \Rightarrow B) \Rightarrow A)$

MP (Rule of inference)

$$(MP) \frac{A ; (A \Rightarrow B)}{B}$$

(1) Prove that H is SOUND under classical semantics.

(2) Does Deduction Theorem holds for H ? Justify shortly your answer.

(3) Is H COMPLETE with respect to all classical semantics tautologies? JUSTIFY your answer.

QUESTION 2 Let H be the proof system defined in QUESTION 1.

(a) Prove the following: $A \vdash_H (A \Rightarrow A)$

(b) We know that $\vdash_H (\neg A \Rightarrow (A \Rightarrow B))$. Prove, that $\neg A, A \vdash_H B$.

QUESTION 3 Here are consecutive steps B_1, \dots, B_5 in the formal proof in H_2 of

$$(B \Rightarrow \neg\neg B)$$

$$B_1 = ((\neg\neg\neg B \Rightarrow \neg B) \Rightarrow ((\neg\neg\neg B \Rightarrow B) \Rightarrow \neg\neg B))$$

$$B_2 = (\neg\neg\neg B \Rightarrow \neg B)$$

$$B_3 = ((\neg\neg\neg B \Rightarrow B) \Rightarrow \neg\neg B)$$

$$B_4 = (B \Rightarrow (\neg\neg\neg B \Rightarrow B))$$

$$B_5 = (B \Rightarrow \neg\neg B)$$

Complete the steps

$$B_1, \dots, B_5$$

of the proof by writing all details in the space provided below each step of the proof.

You have to write down **the proper substitutions and formulas** used at each step of the proof.

You can use the following already proved facts:

1.

$$(A \Rightarrow B), (B \Rightarrow C) \vdash_{H_2} (A \Rightarrow C),$$

2.

$$\vdash_{H_2} (\neg\neg B \Rightarrow B).$$

QUESTION 4 Let A be a formula

$$((-a \Rightarrow \neg b) \Rightarrow c)$$

and let v be such that

$$v(a) = T, \quad v(b) = F, v(c) = F.$$

Evaluate A', B_1, \dots, B_n as defined by the following definition.

Definition Let A be a formula and b_1, b_2, \dots, b_n be all propositional variables that occur in A . Let v be variable assignment $v : VAR \rightarrow \{T, F\}$.

We define, for A, b_1, b_2, \dots, b_n and v a corresponding formulas A', B_1, B_2, \dots, B_n as follows: (for $i = 1, 2, \dots, n$.)

$$A' = \begin{cases} A & \text{if } v^*(A) = T \\ \neg A & \text{if } v^*(A) = F \end{cases}$$

$$B_i = \begin{cases} b_i & \text{if } v(b_i) = T \\ \neg b_i & \text{if } v(b_i) = F \end{cases}$$

QUESTION 5 Consider a system **RS2** obtained from **RS** by changing the sequence Γ' into Γ and Δ into Δ' in all of the rules of inference of **RS**.

1. Construct a decomposition tree in **RS2** of $((a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$

2. Define in your own words, for any A , the decomposition tree \mathbf{T}_A in **RS2**.