

CSE371 Q3 PRACTICE Fall 2009

**QUESTION 1**

$H$  is the following proof system:

$$H = ( \mathcal{L}_{\{\Rightarrow, \neg\}}, \mathcal{F}, AX = \{A1, A2, A3\}, MP )$$

**A1**  $(A \Rightarrow (B \Rightarrow A))$ ,

**A2**  $((A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C)))$ ,

**A3**  $((\neg B \Rightarrow \neg A) \Rightarrow ((\neg B \Rightarrow A) \Rightarrow B))$

**A4**  $((A \Rightarrow B) \Rightarrow A)$

**MP** (Rule of inference)

$$(MP) \frac{A ; (A \Rightarrow B)}{B}$$

- (1) Prove that  $H$  is SOUND under classical semantics.
  
  
  
  
  
  
  
  
  
  
- (2) Does Deduction Theorem holds for  $H$ ? Justify shortly your answer.
  
  
  
  
  
  
  
  
  
  
- (3) Is  $H$  COMPLETE with respect to all classical semantics tautologies?

**QUESTION 2**  $S$  is the following (sound) proof system:

$$S = ( \mathcal{L}_{\{\Rightarrow, \cap\}}, \mathcal{F}, AX = \{A1\} \ \mathcal{R} = \{(r_1), (r_2)\} ),$$

where

**Axiom:**  $A1 = (B \Rightarrow (A \Rightarrow B)),$

**Rules:**

$$(r_1) \frac{A ; B}{(A \cap B)} \qquad (r_2) \frac{A ; (C \cap D)}{(A \Rightarrow (C \cap D))}$$

For the sequence  $B_1, B_2, B_3, B_4$  of formulas of  $\mathcal{L}_{\{\Rightarrow, \cap\}}$  defined below determine if  $B_1, B_2, B_3, B_4$  form a FORMAL PROOF in  $S$ .

If YES, provide comments how each step of the proof was obtained. Write your comments in the space between the steps.

If NOT, write the reason in a proper space between the steps.

$$B_1 = (A \Rightarrow (B \Rightarrow A)),$$

$$B_2 = (B \Rightarrow (A \Rightarrow B)),$$

$$B_3 = ((B \Rightarrow (A \Rightarrow B)) \cap (A \Rightarrow (B \Rightarrow A))),$$

$$B_4 = ((A \Rightarrow (B \Rightarrow A)) \Rightarrow ((B \Rightarrow (A \Rightarrow B)) \cap (A \Rightarrow (B \Rightarrow A))))$$

**QUESTION 3** Let  $H$  be the proof system defined in QUESTION 1.

(a) Prove the following:  $A \vdash_H (A \Rightarrow A)$

(b) We know that  $\vdash_H (\neg A \Rightarrow (A \Rightarrow B))$ . Prove, that  $\neg A, A \vdash_H B$ .

**QUESTION 4** Here are consecutive steps  $B_1, \dots, B_5$  in a proof of  $(B \Rightarrow \neg\neg B)$  in  $H_2$ .

The comments included are incomplete.

**Complete the comments** by writing all details in the space provided below each step of the proof. You have to write down **the proper substitutions and formulas** used at each step of the proof.

$B_1 = ((\neg\neg\neg B \Rightarrow \neg B) \Rightarrow ((\neg\neg\neg B \Rightarrow B) \Rightarrow \neg\neg B))$   
Axiom A3

$B_2 = (\neg\neg\neg B \Rightarrow \neg B)$   
Already proved fact:  $\vdash_{H_2}(\neg\neg B \Rightarrow B)$

$$B_3 = ((\neg\neg\neg B \Rightarrow B) \Rightarrow \neg\neg B)$$

(MP)

$$B_4 = (B \Rightarrow (\neg\neg\neg B \Rightarrow B))$$

Axiom A1

$$B_5 = (B \Rightarrow \neg\neg B)$$

Already proved fact:  $(A \Rightarrow B), (B \Rightarrow C) \vdash_{H_2} (A \Rightarrow C)$