

CSE371 Q4 Practice Fall 2004

NAME

ID:

Math/CS

QUESTION 1 (10pts) Let **GL** be the Gentzen style proof system defined in chapter 10. Prove, by constructing a proper decomposition tree that

(1) (5pts) $\vdash_{\mathbf{GL}}((\neg a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$.

(2) (5pts) $\not\vdash_{\mathbf{GL}}((a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$.

QUESTION 2 (5pt) Does the tree below constitute a proof in **GL**? Justify your answer.

T_{→A}

→ ¬¬((¬a ⇒ b) ⇒ (¬b ⇒ a))

| (→ ¬)

¬((¬a ⇒ b) ⇒ (¬b ⇒ a)) →

| (¬ →)

→ ((¬a ⇒ b) ⇒ (¬b ⇒ a))

| (→ ⇒)

(¬a ⇒ b) → (¬b ⇒ a)

| (→ ⇒)

(¬a ⇒ b), ¬b → a

| (¬ →)

(¬a ⇒ b) → b, a

∧(⇒ →)

→ ¬a, b, a

| (→ ¬)

a → b, a

axiom

b → b, a

axiom

QUESTION 3 (10pts) Let **LI** be the Gentzen system for intuitionistic logic as defined in chapter 11.
Show that

$$\vdash_{\mathbf{LI}} \neg\neg((-a \Rightarrow b) \Rightarrow (-b \Rightarrow a)).$$

QUESTION 4 (Extra 10pts) Use the heuristic method defined in chapter 11 to prove that

$$\not\vdash_{\mathbf{LI}}((\neg a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a)).$$