

CSE371 Q4 Practice Fall 2011

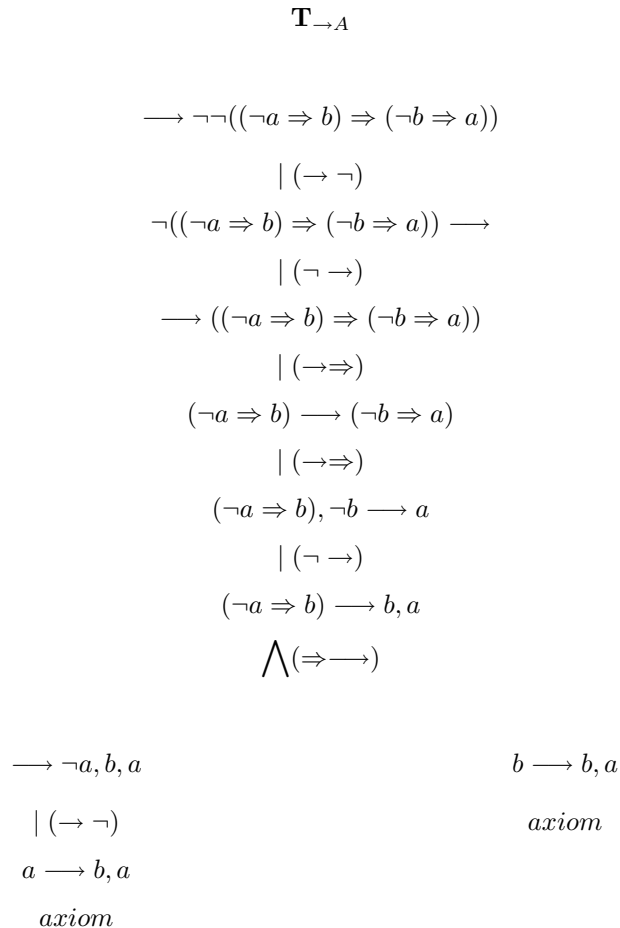
QUESTION 1 Let **GL** be the Gentzen style proof system for classical logic defined in chapter 11. Prove, by constructing a proper decomposition tree that

(1) $\vdash_{\mathbf{GL}}((\neg a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$.

(2) Let **GL** be the Gentzen style proof system defined in chapter 10. Prove, by constructing a proper decomposition tree that

$$\not\vdash_{\mathbf{GL}}((a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a)).$$

QUESTION 2 Does the tree below constitute a proof in **GL**? Justify your answer.



QUESTION 3 Let **GL** be the Gentzen style proof system for classical logic defined in chapter 11. Prove, by constructing a counter-model defined by a proper decomposition tree that

$$\not\models ((a \Rightarrow (\neg b \cap a)) \Rightarrow (\neg b \Rightarrow (a \cup b))).$$

QUESTION 4 Prove the COMPLETENESS theorem for GL. Assume that the Soundness has been already proved and the Decompositions Trees are already defined.

QUESTION 5 Let **LI** be the Gentzen system for intuitionistic logic as defined in chapter 12. Show that

$$\vdash_{\mathbf{LI}} \neg\neg((\neg a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a)).$$

QUESTION 6 We know that the formulas below are not Intuitionistic Tautologies. Verify whether **H** semantics (chapter 5) provides a counter-model for them.

$$((a \Rightarrow b) \Rightarrow (\neg a \cup b))$$

$$((\neg a \Rightarrow \neg b) \Rightarrow (b \Rightarrow a))$$

QUESTION 7 Show that

$$\vdash_{\mathbf{LI}} \neg\neg((\neg a \Rightarrow \neg b) \Rightarrow (b \Rightarrow a))$$

QUESTION 8 Use the heuristic method defined in chapter 12 to prove that

$$\not\models_{\mathbf{LI}}((\neg a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a)).$$

GENERAL REMARK We are using the word "PROOF" in two distinct senses.

In the first sense, we use it as a *formal proof* within a fixed proof system, namely **LI** and is represented as a proof tree, or sequence of expressions of the language \mathcal{L} of **LI**.

In the second sense, it also designates certain sequences of sentences of English language (supplemented by some technical terms, if needed) that are supposed to serve as an argument justifying some assertions about the language \mathcal{L} , or proof system based on it.

In general, the language we are studying, in this case \mathcal{L} , is called *an OBJECT LANGUAGE*.

The language in which we formulate and prove the results about the object language is called *the META-LANGUAGE*. The metalanguage might also be formalized and made the object of study, which we would carry in *a meta-metalanguage*.

We use English as our not formalized metalanguage, although, we use only a mathematically weak portion of the English language. The contrast between the language and metalanguage is also present in study for example, a foreign language. In French study class, French is the object language, while the metalanguage, the language we use, is English.

The distinction between *proof* and *meta-proof*, i.e. a proof in the metalanguage, is now clear. We construct (in the metalanguage) a decomposition tree which is *a formal proof* in the object language. By doing so, we prove in the metalanguage, that the proof in the object language exists. Such proof is called *a meta-proof*, and the fact thus proved is called a *meta-theorem*.