Chapter 1

Basic Simulation Modeling

CONTENTS

1.1 THE NATURE OF SIMULATION

- *Simulation*: Imitate the operations of a facility or process, usually via computer
    - What’s being simulated is the *system*
    - To study system, often make assumptions/approximations, both logical and mathematical, about how it works
    - These assumptions form a *model* of the system
    - If model structure is simple enough, could use mathematical methods to get exact information on questions of interest — *analytical solution*

1.1 The Nature of Simulation (cont’d.)

- But most complex systems require models that are also complex (to be valid)
    - Must be studied via simulation — evaluate model numerically and collect data to estimate model characteristics
- Example: Manufacturing company considering extending its plant
    - Build it and see if it works out?
    - Simulate current, expanded operations — could also investigate many other issues along the way, quickly and cheaply

1.1 The Nature of Simulation (cont'd.)

- Some (not all) application areas
  - Designing and analyzing manufacturing systems
  - Evaluating military weapons systems or their logistics requirements
  - Determining hardware requirements or protocols for communications networks
  - Determining hardware and software requirements for a computer system
  - Designing and operating transportation systems such as airports, freeways, ports, and subways
  - Evaluating designs for service organizations such as call centers, fast-food restaurants, hospitals, and post offices
  - Reengineering of business processes
  - Determining ordering policies for an inventory system
  - Analyzing financial or economic systems

1.1 The Nature of Simulation (cont'd.)

- Use, popularity of simulation
  - Several conferences devoted to simulation, notably the Winter Simulation Conference (www.wintersim.org)
- Surveys of use of OR/MS techniques (examples …)
  - Longitudinal study (1973-1988): Simulation consistently ranked as one of the three most important techniques
  - 1294 papers in *Interfaces* (1997): Simulation was second only to the broad category of "math programming"

1.1 The Nature of Simulation (cont'd.)

- Impediments to acceptance, use of simulation
  - Models of large systems are usually very complex
    - But now have better modeling software … more general, flexible, but still (relatively) easy to use
  - Can consume a lot of computer time
    - But now have faster, bigger, cheaper hardware to allow for much better studies than just a few years ago … this trend will continue
    - However, simulation will also continue to push the envelope on computing power in that we ask more and more of our simulation models
  - Impression that simulation is "just programming"
    - There's a lot more to a simulation study than just "coding" a model in some software and running it to get "the answer"
    - Need careful design and analysis of simulation models – simulation methodology

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1.2 SYSTEMS, MODELS, AND SIMULATION

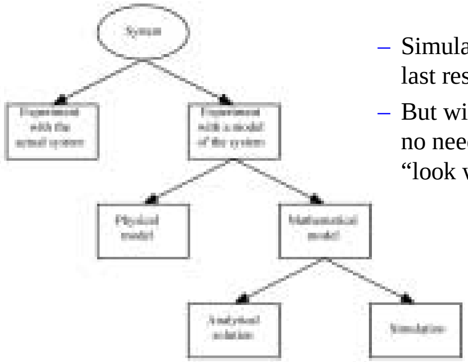
- *System*: A collection of entities (people, parts, messages, machines, servers, …) that act and interact together toward some end (Schmidt and Taylor, 1970)
  - In practice, depends on objectives of study
  - Might limit the boundaries (physical and logical) of the system
  - Judgment call: level of detail (e.g., what is an entity?)
  - Usually assume a time element – *dynamic* system
- *State* of a system: Collection of variables and their values necessary to describe the system at that time
  - Might depend on desired objectives, output performance measures
  - Bank model: Could include number of busy tellers, time of arrival of each customer, etc.

1.2 Systems, Models, and Simulation (cont'd.)

- Types of systems
  - *Discrete*
    - State variables change instantaneously at separated points in time
    - Bank model: State changes occur only when a customer arrives or departs
  - *Continuous*
    - State variables change continuously as a function of time
    - Airplane flight: State variables like position, velocity change continuously
- Many systems are partly discrete, partly continuous

1.2 Systems, Models, and Simulation (cont'd.)

- Ways to study a system



  - Simulation is "method of last resort?" Maybe …
  - But with simulation there's no need (or less need) to "look where the light is"

1.2 Systems, Models, and Simulation (cont'd.)

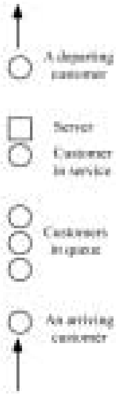
- Classification of simulation models
  - *Static* vs. *dynamic*
  - *Deterministic* vs. *stochastic*
  - *Continuous* vs. *discrete*
- Most operational models are dynamic, stochastic, and discrete – will be called *discrete-event simulation models*

1.3 DISCRETE-EVENT SIMULATION

- *Discrete-event simulation*: Modeling of a system as it evolves over time by a representation where the state variables change instantaneously at separated points in time
  - More precisely, state can change at only a *countable* number of points in time
  - These points in time are when *events* occur
- *Event*: Instantaneous occurrence that may change the state of the system
  - Sometimes get creative about what an "event" is … e.g., end of simulation, make a decision about a system's operation
- Can in principle be done by hand, but usually done on computer

1.3 Discrete-Event Simulation (cont'd.)

- Example: Single-server queue
  - Estimate expected average delay in queue (line, not service)
  - State variables
    - Status of server (idle, busy) – needed to decide what to do with an arrival
    - Current length of the queue – to know where to store an arrival that must wait in line
    - Time of arrival of each customer now in queue – needed to compute time in queue when service starts
  - Events
    - Arrival of a new customer
    - Service completion (and departure) of a customer
    - Maybe – end-simulation event (a "fake" event) – whether this is an event depends on how simulation terminates (a modeling decision)



1.3.1 Time-Advance Mechanisms

- *Simulation clock*: Variable that keeps the current value of (simulated) time in the model
  - Must decide on, be consistent about, time units
  - Usually no relation between simulated time and (real) time needed to run a model on a computer
- Two approaches for time advance
  - *Next-event time advance* (usually used) … described in detail below
  - *Fixed-increment time advance* (seldom used) … Described in Appendix 1A
    - Generally introduces some amount of modeling error in terms of when events *should* occur vs. *do* occur
    - Forces a tradeoff between model accuracy and computational efficiency

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1.3.1 Time-Advance Mechanisms (cont'd.)

- More on next-event time advance
  - Initialize simulation clock to 0
  - Determine times of occurrence of future events – *event list*
  - Clock advances to next (most imminent) event, which is executed
    - Event execution may involve updating event list
  - Continue until stopping rule is satisfied (must be explicitly stated)
  - Clock "jumps" from one event time to the next, and doesn't "exist" for times between successive events … periods of inactivity are ignored

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1.3.1 Time-Advance Mechanisms (cont'd.)

- Next-event time advance for the single-server queue

  t_i = time of arrival of ith customer ($t_0 = 0$)

  $A_i = t_i - t_{i-1}$ = interarrival time between (i-1)st and ith customers (usually assumed to be a random variable from some probability distribution)

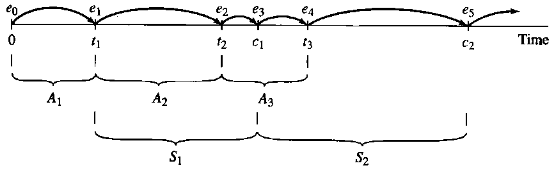
  S_i = service-time requirement of ith customer (another random variable)

  D_i = delay in queue of ith customer

  $C_i = t_i + D_i + S_i$ = time ith customer completes service and departs

  e_j = time of occurrence of the jth event (of any type), $j = 1, 2, 3, \ldots$

  - Possible trace of events (detailed narrative in text)



1.3.2 Components and Organization of a Discrete-Event Simulation Model

- Each simulation model must be customized to target system
- But there are several common components, general organization
  - *System state* – variables to describe state
  - *Simulation clock* – current value of simulated time
  - *Event list* – times of future events (as needed)
  - *Statistical counters* – to accumulate quantities for output
  - *Initialization routine* – initialize model at time 0
  - *Timing routine* – determine next event time, type; advance clock
  - *Event routines* – carry out logic for each event type
  - *Library routines* – utility routines to generate random variates, etc.
  - *Report generator* – to summarize, report results at end
  - *Main program* – ties routines together, executes them in right order

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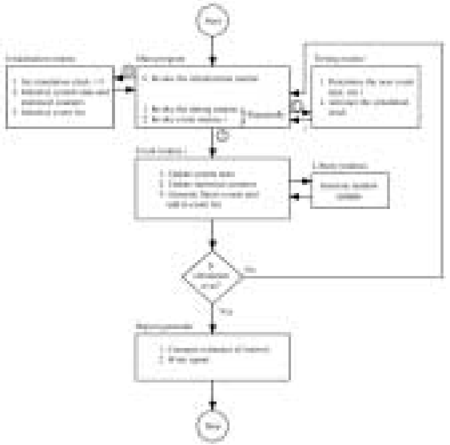
1.3.2 Components and Organization of a Discrete-Event Simulation Model (cont'd.)



1.3.2 Components and Organization of a Discrete-Event Simulation Model (cont'd.)

- More on entities
  - Objects that compose a simulation model
  - Usually include customers, parts, messages, etc. … may include resources like servers
  - Characterized by data values called *attributes*
  - For each entity resident in the model there's a record (row) in a *list*, with the attributes being the columns
- Approaches to modeling
  - *Event-scheduling* – as described above, coded in general-purpose language
  - *Process* – focuses on entities and their "experience," usually requires special-purpose simulation software

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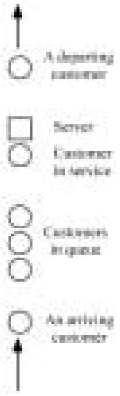
1.4 SIMULATION OF A SINGLE-SERVER QUEUEING SYSTEM

- Will show how to simulate a specific version of the single-server queueing system
- Book contains code in FORTRAN and C … slides will focus only on C version
- Though simple, it contains many features found in all simulation models

1.4.1 Problem Statement

- Recall single-server queueing model
- Assume interarrival times are independent and identically distributed (IID) random variables
- Assume service times are IID, and are independent of interarrival times
- Queue discipline is FIFO
- Start empty and idle at time 0
- First customer arrives after an interarrival time, not at time 0
- Stopping rule: When *n*th customer has completed delay in queue (i.e., *enters* service) … *n* will be specified as input



1.4.1 Problem Statement (cont'd.)

- Quantities to be estimated
  - *Expected average delay in queue* (excluding service time) of the *n* customers completing their delays
    - Why "expected?"
  - *Expected average number of customers in queue* (excluding any in service)
    - A *continuous-time average*
    - Area under $Q(t)$ = queue length at time *t*, divided by $T(n)$ = time simulation ends … see book for justification and details
  - *Expected utilization (proportion of time busy) of the server*
    - Another continuous-time average
    - Area under $B(t)$ = server-busy function (1 if busy, 0 if idle at time *t*), divided by $T(n)$ … justification and details in book
  - Many others are possible (maxima, minima, time or number in system, proportions, quantiles, variances …)
- Important: *Discrete-time* vs. *continuous-time* statistics

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1.4.2 Intuitive Explanation

- Given (for now) interarrival times (all times are in minutes):

  0.4, 1.2, 0.5, 1.7, 0.2, 1.6, 0.2, 1.4, 1.9, …
- Given service times:

  2.0, 0.7, 0.2, 1.1, 3.7, 0.6, …
- *n* = 6 delays in queue desired
- "Hand" simulation:
  - Display system, state variables, clock, event list, statistical counters … all *after* execution of each event
  - Use above lists of interarrival, service times to "drive" simulation
  - Stop when number of delays hits *n* = 6, compute output performance measures

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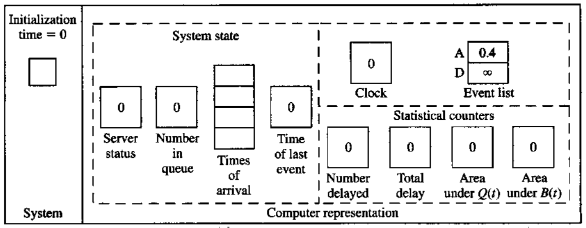
1.4.2 Intuitive Explanation (cont'd)

Status shown is *after* all changes have been made in each case …



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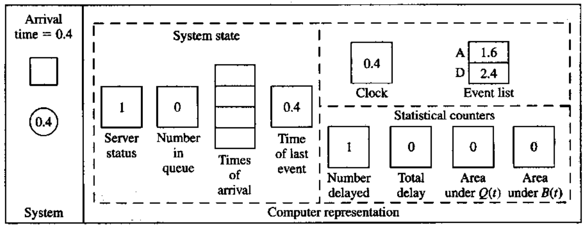
1.4.2 Intuitive Explanation (cont'd)



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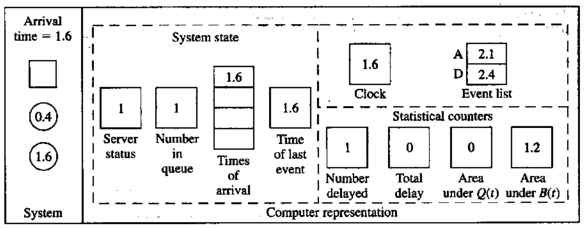

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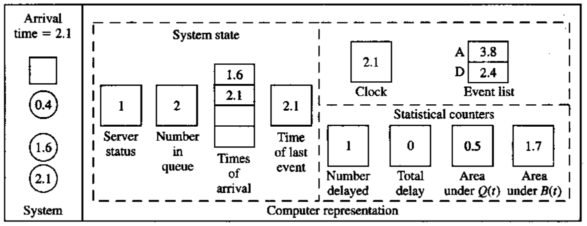

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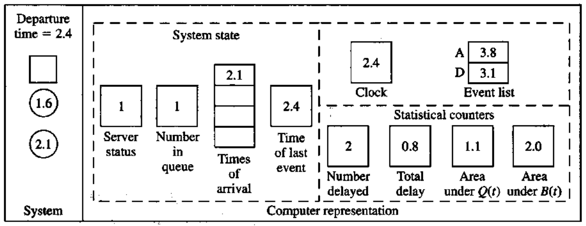

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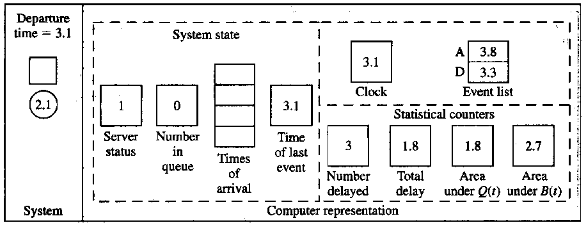

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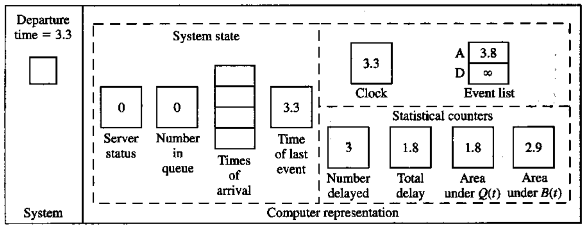

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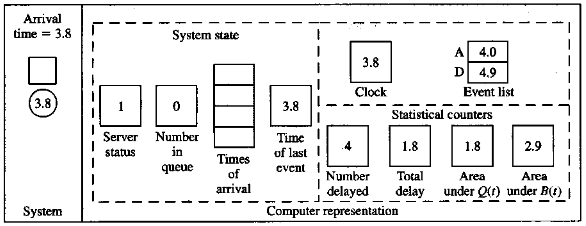

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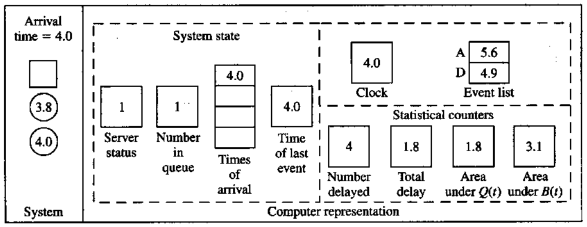

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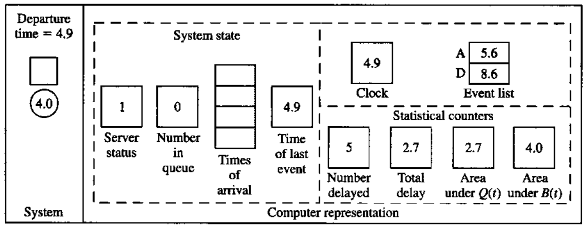

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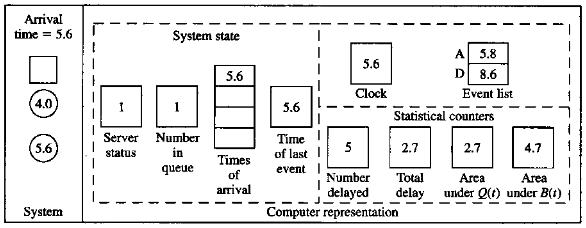

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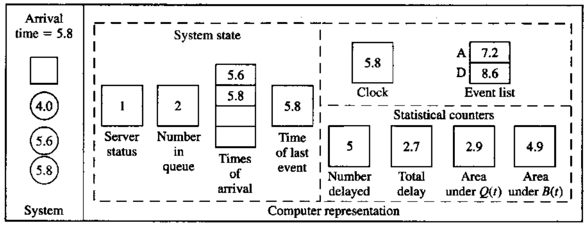

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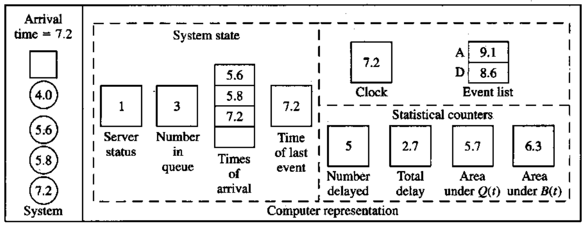

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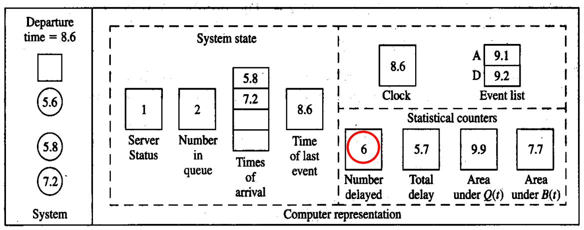

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Service times: ~~2.0~~, ~~0.7~~, ~~0.2~~, ~~1.1~~, ~~3.7~~, ~~0.6~~, …

Final output performance measures:
Average delay in queue = 5.7/6 = 0.95 min./cust.
Time-average number in queue = 9.9/8.6 = 1.15 custs.
Server utilization = 7.7/8.6 = 0.90 (dimensionless)

1.4.3 Program Organization and Logic

- C program to do this model (FORTRAN as well is in book)
  - Event types: 1 for arrival, 2 for departure
  - Modularize for initialization, timing, events, library, report, main
- Changes from hand simulation:
  - Stopping rule: $n = 1000$ (rather than 6)
  - Interarrival and service times "drawn" from an exponential distribution (mean $\beta = 1$ for interarrivals, 0.5 for service times)
    - Density function $f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$
    - Cumulative distribution function

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt = \begin{cases} 1 - e^{-x/\beta} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

1.4.3 Program Organization and Logic (cont'd.)

- How to "draw" (or generate) an observation (*variate*) from an exponential distribution?
- Proposal:
  - Assume a perfect *random-number generator* that generates IID variates from a continuous uniform distribution on [0, 1] … denoted the U(0, 1) distribution … see Chap. 7
  - Algorithm:
    1. Generate a random number *U*
    2. Return $X = -\beta \ln U$
  - Proof that algorithm is correct: $P(\text{generated } X \leq x) = P(-\beta \ln U \leq x)$

$$\begin{aligned} &= P(\ln U \geq -x/\beta) \\ &= P(U \geq e^{-x/\beta}) \\ &= P(e^{-x/\beta} \leq U \leq 1) \\ &= 1 - e^{-x/\beta} \end{aligned}$$

1.4.5 C Program;
1.4.6 Simulation Output and Discussion

- Refer to pp. 30, 31, 42-48 in the book (Figures 1.8, 1.9, 1.19-1.27) and the file **`mm1.c`**
  - Figure 1.19 – external definitions (at top of file)
  - Figure 1.20 – function **`main`**
  - Figure 1.21 – function **`initialize`**
  - Figure 1.22 – function **`timing`**
  - Figure 1.23 – function **`arrive`** (flowchart: Figure 1.8)
  - Figure 1.24 – function **`depart`** (flowchart: Figure 1.9)
  - Figure 1.25 – function **`report`**
  - Figure 1.26 – function **`update_time_avg_stats`**
  - Figure 1.27 – function **`expon`**
  - Figure 1.28 – output report **`mm1.out`**
    - Are these "the" answers?
    - Steady-state vs. terminating?
    - What about time in queue vs. just time in system?

1.4.7 Alternative Stopping Rules

- Stop simulation at (exactly) time 8 hours (= 480 minutes), rather than whenever *n* delays in queue are completed
  - Before, final value of simulation clock was a random variable
  - Now, number of delays completed will be a random variable
- Introduce an artificial “end-simulation” event (type 3)
  - Schedule it on initialization
  - Event routine is report generator
  - Be sure to update continuous-time statistics to end
- Changes in C code (everything else is the same)
  - Figure 1.33 – external definitions
  - Figure 1.34 – function **`main`**
  - Figure 1.35 – function **`initialize`**
  - Figure 1.36 – function **`report`**
  - Figure 1.37 – output report **`mm1alt.out`**

1.4.8 Determining the Events and Variables

- For complex models, it might not be obvious what the events are
- *Event-graph* method (Schruben 1983, and subsequent papers) gives formal graph-theoretic method of analyzing event structure
- Can analyze what needs to be initialized, possibility of combining events to simplify model
- Software package (SIGMA) to build, execute a simulation model via event-graph representation

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1.5 SIMULATION OF AN INVENTORY SYSTEM;
1.5.1 Problem Statement

- Single-product inventory
- Decide how many items to have in inventory for the next *n* = 120 months; initially (time 0) have 60 items on hand
- Demands against inventory
  - Occur with inter-demand time ~ exponential with mean 0.1 month
  - Demand size = 1, 2, 3, 4 with resp. probabilities 1/6, 1/3, 1/3, 1/6
- Inventory review, reorder – stationary (*s*, *S*) policy … at beginning of each month, review inventory level = *I*
  - If $I \geq s$, don’t order (*s* is an input constant); no ordering cost
  - If $I < s$, order $Z = S - I$ items (*S* is an input constant, order “up to” *S*); ordering cost = 32 + 3*Z*; delivery lag ~ U(0.5, 1) month

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1.5.1 Problem Statement (cont’d.)

- Demand in excess of current (physical) inventory is backlogged … so (accounting) inventory could be < 0
- Let *I*(*t*) be (accounting) inventory level at time *t* (+, 0, –)

  $I^+(t) = \max\{I(t), 0\}$ = number of items physically on hand at time *t*

  $I^-(t) = \max\{-I(t), 0\}$ = number of items in backlog at time *t*
- *Holding cost*: Incur $1 per item per month in (positive) inventory

  Time-average (per month) holding cost = $\$1\int_0^n I^+(t)\,dt / n$
- *Shortage cost*: Incur $5 per item per month in backlog

  Time-average (per month) backlog cost = $\$5\int_0^n I^-(t)\,dt / n$
- Average total cost per month: Add ordering, holding, shortage costs per month
  - Try different (*s*, *S*) combinations to try to reduce total cost

1.5.2 Program Organization and Logic

- State variables: Inventory level, amount of an outstanding order, time of the last (most recent) event
- Events:
  1. Arrival of an order from the supplier
  2. Demand for the product
  3. End of the simulation after $n = 120$ months
  4. Inventory evaluation (maybe ordering) at beginning of a month

Why the ordering of event types 3 and 4?

- Random variates needed
  - Interdemand times: exponential, as in queueing model
  - Delivery lags ~ U(0.5, 1): $0.5 + (1 - 0.5)U$, where $U \sim U(0, 1)$
  - Demand sizes: Split [0, 1] into subintervals of width 1/6, 1/3, 1/3, 1/6; generate $U \sim U(0, 1)$; see which subinterval U falls in; return $X = 1, 2, 3$, or 4, respectively

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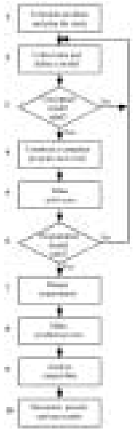
1.5.4 C Program;
1.5.5 Simulation Output and Discussion

- Refer to pp. 64-66, 73-79 in the book (Figures 1.43-1.46, 1.57-1.67) and the file **`inv.c`**
  - Figure 1.57 – external definitions (at top of file)
  - Figure 1.58 – function **`main`**
  - Figure 1.59 – function **`initialize`**
  - Figure 1.60 – function **`order_arrival`** (flowchart: Figure 1.43)
  - Figure 1.61 – function **`demand`** (flowchart: Figure 1.44)
  - Figure 1.62 – function **`evaluate`** (flowchart: Figure 1.45)
  - Figure 1.63 – function **`report`**
  - Figure 1.64 – function **`update_time_avg_stats`** (flowchart: Figure 1.46)
  - Figure 1.65 – function **`random_integer`**
  - Figure 1.66 – function **`uniform`**
  - Figure 1.67 – output report **`inv.out`**
    - Reaction of individual cost components to changes in s and S … overall?
    - Uncertainty in output results (this was just one run)?

1.6 ALTERNATIVE APPROACHES TO MODELING AND CODING SIMULATIONS

- Parallel and distributed simulation
  - Various kinds of parallel and distributed architectures
  - Break up a simulation model in some way, run the different parts simultaneously on different parallel processors
  - Different ways to break up model
    - By support functions – random-number generation, variate generation, event-list management, event routines, etc.
    - Decompose the model itself; assign different parts of model to different processors – message-passing to maintain synchronization, or forget synchronization and do “rollbacks” if necessary … “virtual time”
- Web-based simulation
  - Central simulation engine, submit “jobs” over the web
  - Wide-scope parallel/distributed simulation

1.7 STEPS IN A SOUND SIMULATION STUDY



1.8 OTHER TYPES OF SIMULATION

- *Continuous simulation*
  - Typically, solve sets of differential equations numerically over time
  - May involve stochastic elements
  - Some specialized software available; some discrete-event simulation software will do continuous simulation as well
- *Combined discrete-continuous simulation*
  - Continuous variables described by differential equations
  - Discrete events can occur that affect the continuously-changing variables
  - Some discrete-event simulation software will do combined discrete-continuous simulation as well

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1.8 Other Types of Simulation (cont'd.)

- *Monte Carlo simulation*
  - No time element (usually)
  - Wide variety of mathematical problems
  - Example: Evaluate a "difficult" integral $I = \int_a^b g(x)\,dx$
    - Let $X \sim U(a, b)$, and let $Y = (b - a)\, g(X)$
    - Then $E(Y) = E[(b-a)g(X)]$

$$
\begin{aligned}
&= (b-a)E[g(X)] \\
&= (b-a)\int_a^b g(x) f_X(x)\,dx \\
&= (b-a)\int_a^b g(x)\frac{1}{b-a}\,dx \\
&= \int_a^b g(x)\,dx \\
&= I
\end{aligned}
$$

    - Algorithm: Generate $X \sim U(a, b)$, let $Y = (b - a)\, g(X)$; repeat; average the Y's … this average will be an unbiased estimator of I

1.9 ADVANTAGES, DISADVANTAGES, AND PITFALLS OF SIMULATION

- Advantages
  - Simulation allows great flexibility in modeling complex systems, so simulation models can be highly valid
  - Easy to compare alternatives
  - Control experimental conditions
  - Can study system with a very long time frame
- Disadvantages
  - Stochastic simulations produce only estimates – with noise
  - Simulation models can be expensive to develop
  - Simulations usually produce large volumes of output – need to summarize, statistically analyze appropriately
- Pitfalls
  - Failure to identify objectives clearly up front
  - In appropriate level of detail (both ways)
  - Inadequate design and analysis of simulation experiments
  - Inadequate education, training