CHAPTER 1

INTRODUCTION

1 Mathematical Paradoxes

Until recently, till the end of the 19th century, mathematical theories used to be built in an intuitive or axiomatic way. In other words, they were based either intuitive ideas concerning basic notions of the theory - ideas taken from the reality- or on the properties of these notions expressed in systems of axioms. The historical development of mathematics has shown that it is not sufficient to base theories on an intuitive understanding of their notions only. This fact became especially obvious in set theory. The basic concept of this theory, *set*, is certainly taken from reality, for there we come across many examples of various sets, all of which are finite. But in mathematics it is also necessary to consider infinite sets, such as the set of all integers, the set of all rational numbers, the set of all segments, the set of all triangles.

By a set, we mean intuitively, any collection of objects- for example, the set of all even integers or the set of all students in a class. The objects that make up a set are called its members (elements). Sets may themselves be members of sets for example, the set of all sets of integers has sets as its members. Most sets are not members of themselves; the set of all students, for example, is not a member of itself, because the set all students is not a student. However, there may be sets that do belong to themselves - for example, the set of all sets. However, a simple reasoning indicates that it is necessary to impose some limitations on the concept of a set.

Russell, 1902 Consider the set A of all those sets X such that X is not a member of X. Clearly, by definition, A is a member of A if and only if A is not a member of A. So, if A is a member of A, the A is also not a member of A; and if A is not a member of A, then A is a member of A. In any case, A is a member of A and A is not a member of A.

This paradox arises because the concept of set was not precisely enough defined, and was too liberally interpreted. Russell noted the self-reference present in his paradox (and other paradoxes, two of them stated below) and suggested that every object must have a definite non-negative integer as its *type*. Then an expression *x is a member of the set y* is *meaningful* if and only if the type of *y* is one greater than the type of *x*. So, according to the theory of types, it is meaningless to say that a set belongs to itself, there can not be such set A, as stated in the Russell paradox.

The paradoxes concerning the notion of a set are called *logical paradoxes (anti-nomies)*. Two of the most known (besides the Russell's) *logical paradoxes* are Cantor and Burali-Forti antinomies. Both were stated at the end of 19th century. The Cantor paradox involves the theory of *cardinal numbers*, Burali-Forti paradox is the analogue to Cantor's in the theory of *ordinal numbers*. They will make real sense only to those already familiar with both of the theories, but we will state them here because they do have an intuitive meaning and had very important consequences.

The cardinal number $cardX$ of a set X intuitively corresponds, in a case of finite sets, to a *number of elements of the set X. cardX* is formally defined to be the set of all sets Y that are *equinumerous* with X (i.e., for which there a one-to-one correspondence between X and Y). We define $cardX \leq cardY$ to mean that X is equinumerous with a subset of Y; by $cardX < cardY$ we mean $cardX \leq cardY$ and $cardX \neq cardY$. Cantor proved that *if $\mathcal{P}(X)$ is the set of all subsets of X, then $cardX < card\mathcal{P}(X)$*. The cardinal numbers behave similarly to natural numbers in many cases, in particular Schröder- Berstein proved that *if $cardX \leq cardY$ and $cardY \leq cardX$, then $cardX = cardY$*.

The *ordinal numbers* are the numbers assigned to sets in a similar way as cardinal numbers but they deal with *ordered* sets.

Cantor, 1899 Let C be the universal set - that is, the set of all sets. Now, $\mathcal{P}(C)$ is a subset of C, so it follows easily that $card\mathcal{P}(C) \leq cardC$. On the other hand, by Cantor theorem, $cardC < card\mathcal{P}(C)$, so also $cardC \leq card\mathcal{P}(C)$ and by Schröder- Berstein theorem we have that $card\mathcal{P}(C) = cardC$, what contradicts $cardC < card\mathcal{P}(C)$.

Burali-Forti, 1897 Given any ordinal number, there is a still larger ordinal number. But the ordinal number determined by the set of all ordinal numbers is the largest ordinal number.

The approach of eliminating logical paradoxes, known as the theory of types was systematized and developed by Whitehead and Russell in years 1910 - 1913. It is successful, but difficult in practice and has certain other drawbacks as well. A different criticism of the logical paradoxes is aimed at their assumption that the notion of a set is defined in such a way that, for every property $P(x)$, there exists a corresponding set of all objects x that satisfy $P(x)$. If we reject this assumption, then the logical paradoxes are no longer derivable. Russell's Paradox then simply proves that there is no set A of all sets that do not belong to themselves; the paradoxes of Cantor and Burali-Forti show that there is no universal set and no set that contains all ordinal numbers.

It became obvious that the paradoxes described above, as well as many similar ones occurring in intuitive set theory cannot be avoided by referring to intuition only. The solution looked for was to characterize the intuitive concept of set by

a suitable set of *axioms*. If we do so, we obtain an *axiomatic set theory* without such antinomies. The problem arises what set of axioms should be chosen in order to obtain a sufficiently rich theory of sets. The first such axiomatic set theory was invented by Zermello in 1908. In chapter .. we shall present an axiomatic theory of sets that is descendant of Zermello' system. The theory based on that system is, in practice, sufficient to include all actual mathematical theories!

A more radical interpretation of the paradoxes has been advocated by Brower and his *intuitionist* school. They refuse to accept the universality of certain basic logical laws, such as the law of *excluded middle*: A or not A. Such a law, they claim, is true for finite sets, but it is invalid to extend it to all sets. It means that the intuitionists' view of the concept of infinite set differs from that of most mathematicians. Intuitionists reject the idea of infinite set as a closed whole. They look upon an infinite set as something which is constantly in a state of formation. It means that, for example, the set of all positive integers is not looked upon as a closed whole. It is infinite in a sense that to any given finite set of positive integers it is always possible to add one more positive integer. The notion of the set of all subsets of the set of all positive integers is not regarded meaningful. Obviously, intuitionists' view-point on the meaning of the basic logical and set-theoretical concepts used in mathematics is different from that of most mathematicians in their research. The basic difference lies in the interpretation of the word *exists*. For example, let $P(n)$ be a statement in the arithmetic of positive integers. For the mathematician the sentence *the exists n, such that P(n)* is true if it can be *deduced* (proved) from the axioms of arithmetic by means of classical logic. If the mathematician proves it, this does not mean that he is able to indicate a method of *construction* of a positive integer n such that $P(n)$ holds. On the contrary, for the intuitionist the sentence *the exists n, such that P(n)* is true only if he is able to *construct* a number n such that $P(n)$ is true. In the intuitionists' universe we are justified in asserting the existence of an object having a certain property only if we know an effective method for constructing or finding such an object. The paradoxes are, in this case, not derivable (or even meaningful), but so are many theorems of everyday mathematics, and for this reason, intuitionism has found few converts among mathematicians. But, because of its constructive flavor, it has found some applications in computer science and this is the reason to discuss some of it here. An exact exposition of the basic ideas of intuitionism is outside the range of this book, but we will study *intuitionists logic*, which is a sort of reflection of intuitionists ideas formulated in formalized deductive system.

As we can see, the axiomatic method was the first step leading to greater precision in the construction of mathematical theories. In intuitive mathematical theories the borderline between that which is obvious and that which requires proof is not exact. In axiomatic theories a system of primitive notions is assumed which are characterized by a set of axioms. Other notions can be defined by means of the primitive notions. All statements which are consequences of

the axioms are called theorems of the theory. All properties of any notion of the theory which are not expressed in the axioms, require proof.

For some time this degree of exactness in the construction of theories seemed sufficient. However, it turned out that the assumption of a consistent set of axioms does not prevent the occurrence of another kind of paradoxes, called *semantic paradoxes*.

For instance, let us consider the arithmetic based on the well known system of axioms due to Peano (to be discussed in chapter...) and let's consider the following simple reasoning.

Berry, 1906 Let A denote the set of all positive integers which can be defined in the English language by means of a sentence containing at most 1000 letters. The set A is finite since the set of all sentences containing at most 1000 letters is finite. Hence, there exist positive integers which do not belong to A. The sentence:

n is the least positive integer which cannot be defined by means of a sentence of the English language containing at most 1000 letters

contains less than 1000 letters and defines a positive integer n. Therefore n belongs to A. On the other hand, n does not belong to A by the definition of n. This contradicts the first statement.

It is obvious that the reason for this paradox is that in its construction we used some notions (e.g the English language, letters, sentences) which do not belong to pure arithmetic. Usually we do not introduce definitions like the above in mathematics. The paradox resulted entirely from the fact that we did not say precisely what notions and sentences *belong* to the arithmetic and what notions and sentences *concern* the arithmetic, examined as a fix and closed deductive system. Intuitively we conceive the arithmetic to be a set of sentences expressing certain properties of positive integers and of other notions defined by means of the notion of integer. For example, a sentence: *for every integer n, $n^2 \leq 0$* certainly belongs to arithmetic.

On the other hand we can also talk *about* the arithmetic. That is, assuming that all sentences in the arithmetic are formulated in the English language, we can formulate sentences concerning the problem how many integers we defined in the arithmetic by means of at most 1000 letters. However, such sentences about the arithmetic do not *belong* to arithmetic. They belong to another theory, which *examines* the arithmetic as a new subject of investigation. This theory is usually called *meta-arithmetic*. In particular, the Berry sentence does not belong to arithmetic; it belongs to meta-arithmetic and the paradox arises from the luck of distinction between a theory (language) and its meta-theory (metalanguage).

For a similar reason in well defined theory the following paradoxes can not appear.

The Liar Paradox (Greek philosopher Eubulides of Miletus, 400 BC)
>   A man says: *I am lying.* If he is lying, then what he says is true, and so he is not lying. If he is not lying, then what he says is not true, and so he is lying. In any case, he is lying and he is not lying.

Löb, 1955 Let A be any sentence. Let B be a sentence: *If this sentence is true, then A.* So, B asserts: *If B is true then A.* Now consider the following argument: Assume B is true. Then, by B, since B is true, A is true. This argument shows that, if B is true, then A. But this is exactly what B asserts. Hence, B is true. Therefore, by B, since B is true, A is true. Thus every sentence is true.

In these cases the paradox arises because the concepts of " I am true", " this sentence is true, " I am lying"" should not occur in the language (theory). It belong to a metalanguage (meta-theory).

The Liar Paradox is a corrected version of a following paradox stated in antiquity by a Cretan philosopher Epimenides, 600 BC.

Cretan " Paradox" (The Cretan philosopher Epimenides paradox, 600 BC)

>   Epimenides, a Cretan said: *All Cretans are liars.* If what he said is true, then, since Epimenides is a Cretan, it must be false. Hence, what he said is false. Thus, there must be some Cretan who is not a liar.

Note that the conclusion that there must be some Cretan who is not a liar is not logically impossible, so we do not have a genuine paradox. However, the fact that the utterance by Epimenides of the false sentence could imply the existence of some Cretan who is not a liar is rather unsettling.

It follows from above *semantic paradoxes* that in order to exclude them from an axiomatic theory, it is necessary to describe its *language* precisely, i.e. the set of sentences of the theory and the set of signs used to build these sentences. In this way we avoid contradictions caused by a collision between the theory and meta-theory, that is, by including meta-theoretical statements in the theory. This inclines us to introduce still greater precision in the construction of mathematical (and others) theories and leads to the concept of *formalized theories*, in which not only the properties of primitive notions are given in an exact way by means of a set of *axioms*, but also the *language of the theory* is precisely defined. The formalization of the language also gives the following advantage: it permits us to describe precisely the logical means assumed in the theory, i.e. to define the process of *deduction* precisely.

In formalized mathematical theories, e.g. in formalized arithmetic and formalized set theory, the paradoxes as those mentioned above cannot be constructed. On the other hand, a mathematician (or a computer scientist) following good intuitions in every -day investigations does not reach a contradiction even though the language of the theory and the logical means he employs are not precisely described. This is explained by the fact that his investigations can always in practice be repeated in a corresponding formalized theory. Thus he avoids practical difficulties of formalized theories, the formalized language of of which is complicated and very inconvenient in every day practice. Consequently, in mathematical practice we build theories axiomatically but always in such a way that they can be easily formalized, i.e. that all the reasonings can be repeated in a corresponding formalized theory. However, the formalization of the language and the logical means are necessary if we intend to develop the meta-theory of a given practical theory because only in this way such important notions as the existence of a proof of a given statement or the set of all theorems of the theory can be *precisely* defined. In practical, non-formalized axiomatic theories those notions are far from this precision.

Whatever approach one takes to the paradoxes, it is always necessary first to examine the language of logic and mathematics or given domain of computer science, to see what *symbols* may be used, to determine the way ways in which these symbols are put together to form terms, formulas, sentences, and proofs, and to find out what can and cannot be proved if certain axioms and rules of inference are assumed.

This is the basic task of mathematical logic, and, until it is done, there is no basis of talking about foundations of logic, mathematics or computer science.

This approach is already almost a century old - the first formalized theory was built by Frege in 1891. The deep and devastating results of Gödel, Tarski, Church, Rosser, Kleene and many others followed. They created what is called a modern mathematical logic and have earned for it its status as an independent branch of mathematics.

2 Computer Science Puzzles

Logical and semantical paradoxes have lead the mathematicians to the development of a modern classical logic as an independent domain of mathematics. They have also, as we could see, lead to the development of the intuitionistic logic as rival to the classical one. The classical and intuitionistic logic differ on the interpretation of the meaning of the word *exists*, but also, as we will see later, in the interpretation of logical implication, i.e. the truthfulness of the sentences of the form *if A then B* is decided differently in both logics.

In 1918, an American philosopher, C.I. Lewis proposed yet another interpreta-

tion of lasting consequences, of the logical implication. In an attempt to avoid, what some felt, the paradoxes of implication (a false sentence implies any sentence) he created a *modal logic*. The idea was to distinguish two sorts of truth: *necessary* truth and mere *possible (contingent)* truth. A *possibly* true sentence is one which, though true, could be false.

More recently, modal logic has become much-used in computer science as a tool for analyzing such notions as knowledge, belief, tense.

The logics other than the classical propositional or predicate logic are usually called *non-standard logics*. The use of classical logic on computer science is known, undisputable, and well established.The existence of PROLOG and Logic Programming as a separate field of computer science is the best example of it. But the non-standard logics have been imported into a great many areas of computer science and, in particular into the research about the specification and verification of programs, the analysis of behavior of distributed systems and into almost all areas of artificial intelligence. Even in Logic Programming, once we start to talk *about* logic programming programs we run immediately into some non-standard logics.

Modal logic, for example, has been employed in form of *Dynamic logic* (Harel 1979) to facilitate the statement and proof of properties of programs.

Temporal Logics were created for the specification and verification of concurrent programs Harel, Parikh, 1979, 1983), for a specification of hardware circuits Halpern, Manna and Maszkowski, (1983), to specify and clarify the concept of causation and its role in commonsense reasoning (Shoham, 1988).

Intuitionistic logic, in the form of Martin-Löf's theory of types (1982), provides a complete theory of the process of program specification, construction , and verification. A similar theme has been developed by Constable (1971) and Beeson (1983).

The great strength of dynamic and temporal logics relates to their expressive power. In such systems it is possible to express properties of programs in an elegant and natural way. This is in large part due to enriched language of such logics over the classical predicate calculus. In the case of intuitionistic logic the motivation for their employment, as it was mentioned before, is different. The proponents of intuitionistic logic and mathematics claim that *constructive* mathematics is, generally, a more appropriate framework for computer science than classical logic and mathematics.

Fuzzy logic, Many valued logics were created and developed to reasoning with incomplete information. Most expert systems are forced to take decisions when not all the facts pertaining to decision are available. In such context it is natural to employ logics which, unlike classical logic, are suited to reasoning with such incomplete information.

The development of different logics and the applications of logic to different areas of computer science or even artificial intelligence only is far beyond the scope of class.

We will present now some the most known motivations (*computer science puzzles*), which have played lately a similar role in the development of the reasoning about knowledge in *distributed systems* and *artificial intelligence* as logical and semantical paradoxes have played in the development of logic and foundations of mathematics.

2.1 Reasoning about knowledge in distributed systems

The major complexities in designing, understanding and reasoning about distributed systems arise from the uncertainties inherent in the system, particularly with regard to message delivery and possible faulty or unexpected behavior of processors. A protocol must be designed (and proved!) to function properly even if it is possible for messages to be lost, for messages to arrive out of order, or for some processor to fail. It is difficult (probably) impossible for one node to *know* everything about the rest of the network. Yet we are often interested in situations in which everyone in a group (every processor in the network) knows a certain fact. But even the state of knowledge in which everyone knows that everyone knows does not suffice for a number of applications. In some cases we also need to consider the state in which simultaneously everyone knows a fact F, everyone knows that everyone knows F, everyone knows that everyone knows that everyone knows F, and so on. In this case it is said that the group has *common knowledge* of F. The relationship between the common knowledge, simultaneous agreement and coordinate action is nicely put together in the *coordinated attack problem*, from the distributed system folklore.

Grey, 1978. Halpern, Moses, 1984 Two divisions of an army are camped on two hilltops overlooking a common valley. In the valley awaits the enemy. It is clear that if both divisions attack the enemy simultaneously they will win the battle, whereas if only one division attacks it will be defeated. The divisions do not initially have plans for launching an attack on the enemy, and the commanding general of the first division wishes to coordinate a simultaneous attack (at some time the next day). Neither general will decide to attack unless he is sure that the other will attack with him. The generals can only communicate by means of a messenger. Normally, it takes a messenger one hour to get from one encampment to the other. However, it is possible that he will get lost in the dark or, worst yet, be captured by the enemy. Fortunately, on this particular night, everything goes smoothly. How long will it take them to coordinate an attack?

Suppose the messenger sent by General A makes it to General B with a message saying *Attack at dawn.* Will B attack? No, since A does not know B got the message, and thus may not attack. So B sends the messenger back with an acknowledgment. Suppose the messenger makes it. Will A attack? No, because now A is worried that B does not know A got the message, so that B thinks A may think that B did not get the original message, and thus not attack. So A sends the messenger back with an acknowledgment. But of course, this is not enough either. It is not difficult to be convinced that no amount of acknowledgments sent back and forth will ever guarantee agreement. Even in a case that the messenger succeeds in delivering the message every time. All that is required in this (informal) reasoning is the possibility that the messenger doesn't succeed.

This rather convoluted reasoning was expressed formally by Halpern and Moses in 1985 in terms of *a propositional modal logic with m agents.* They proved this logic to be essentially a multi-agent version of the *modal logic S5.*, which we will study in chapter ....

They also showed that not only is common knowledge (formally defined!) not attainable in systems where communication is not guaranteed, it is also not attainable in systems where communication *is* guaranteed, as long as there is some uncertainty in massage delivery time. Thus, in practical distributed systems , common knowledge is not attainable. This holds for systems of communicating humans as well as processors. What is going on here? After all, we often do reach agreement! It was shown that common knowledge (as formally defined) is attainable in such models of reality where we assume, for example, events can be guaranteed to happen simultaneously. It turns also out that even we can't always make this assumption in practice, there are some variants of the definition of common knowledge that are attainable under more reasonable assumptions, and these variants are indistinguishable in certain cases from the "true" common knowledge, as originally defined.. So, finally, we can *prove* that in fact we often do reach agreement!

2.2   Reasoning in Artificial Intelligence

A key property of intelligence, one may agree, is *flexibility.* This flexibility is intimately connected with the defeasible nature of commonsense inference; we are all capable of drawing conclusions, acting on them, and then retracting them if necessary in the face of new evidence. If our computer programs are to act intelligently, they will need to be similarly flexible.

A large portion of the work in artificial intelligence (AI) on reasoning or deduction involves the development of formal systems that describe this process.

The most usual example of such a flexible inference is the following flying birds

example.

Reiter, 1987 Consider a statement *Birds fly*. Tweety, we are told, is a bird. From this, and the fact that birds fly, we conclude that Tweety can fly.

This conclusion, however is *defeasible*: Tweety may be an ostrich, a penguin, a bird with a broken wing, or a bird whose feet have been set in concrete.

The inference here is *non-monotonic*: on learning a new fact (that Tweety has a broken wing), you are forced to retract your conclusion that he could fly. This original conclusion didn't follow logically (in a sense if classical logic) from the facts that birds typically fly and that Tweety is a bird; it followed from these facts *together* with the *assumption* that Tweety is a typical bird. When we learn more about Tweety, we may discover that this assumption is unjustified.

It means, by a *non-monotonic reasoning* (logics) we understand reasonings (logics) in which the introduction of a new information (axioms) can invalidate old theorems.

The inference described here is also called a *default reasoning*.

It means, by *default reasoning* we mean the drawing of plausible inferences from less-then- conclusive evidence in the absence of information to the contrary.

Consider now the following example.

Moore, 1983 Consider my reason for believing that I do not have an older brother. It is surely not that one of my parents once casually remarked, *You know, you don't have any older brothers*, nor have I pieced it together by carefully sifting other evidence. I simply *believe* that if I did have an older brother I would know about it; therefore, since I don't know of any older brothers of mine, I must not have any.

This type of reasoning is not a form of default reasoning at all; it rather seems to be more like reasoning about one's own knowledge or belief. Hence it is called an *auto-epistemic* reasoning.

It means, by *auto-epistemic reasoning* we mean the reasoning about one's own beliefs or knowledge.

The auto-epistemic reasoning is intended to model the reasoning of an ideally rational agent reflecting upon his beliefs or knowledge. Logics which describe it are called *auto-epistemic logics*.

In addition to application to the understanding of common-sense reasoning, non-monotonic reasoning (*non-monotonic logics*) has been shown to be important in

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other areas. There are applications to logic programming, to planning and reasoning about action, and to automated diagnosis. As the formal work matures, increasing effort is being devoted to applying the improved understanding to the solution of practical problems.

We will end this introduction by McCarthy discussion of a much used in AI puzzle *Missionaries and Cannibals*, as a proof of a need of another "stretch " from classical logic.

McCarthy, 1985 Consider the *Missionaries and Cannibals* puzzle.

> *Three missionaries and three cannibals come to the river. A rowboat that seats two is available. If the cannibals ever outnumber the missionaries on either bank of the river, the missionaries will be eaten. How shall they cross the river?*

Obviously the puzzler is expected to devise a strategy of rowing the boat back and forth that gets them all across and avoids the disaster.

Ammarel considered several representations of the problem and discussed criteria whereby the following representation is preferred for purposes of AI, because it leads to the smallest state space that must be explored to find the solution. A state is a triple comprising the number of missionaries, cannibals and boats on the starting bank of the river. The initial state is 331, the desired state is 000, and one solution is given by the sequence: 331, 220, 321, 300,311, 110, 221, 020, 031, 010, 021, 000.

We are not presently concerned with the heuristic of the problem but rather with the *correctness of the reasoning* that goes from the English statement of the problem to Amerel's state space representation. A generally intelligent computer program should be able to carry out this reasoning. Of course, there are the well known difficulties in making computers understanding English, but suppose the English sentences describing the problem have already been rather directly translated into first order logic. The correctness of Amarel's representation is not an ordinary logical consequence of these sentences for two further reasons. First, nothing has been said about the properties of boats or even the fact that rowing across the river doesn't change the number of missionaries or cannibals or the capacity of the boat. Indeed it hasn't been stated that situations change as a result of action. These facts follow from common sense knowledge, so let us imagine that common sense knowledge, or at least the relevant part of it, is also expressed in first order logic.

The second reason we can't *deduce* the propriety of Amarel's representation is deeper. Imagine giving someone a problem, and after he puzzles for a while, he suggests going upstream half a mile and crossing on a bridge. *What a bridge?* you say. *No bridge is mentioned in the statement of the problem.* And this dunce replies, *Well, they don't say the isn't a bridge.*

You look at the English and even at the translation of the English into first order logic, and you must admit that *they don't say* there is no bridge. So you modify the problem to exclude the bridges and pose it again, and the dunce proposes a helicopter, and after you exclude that, he proposes a winged horse or that the others hang onto the outside of the boat while two row.

You now see that while a dunce, he is an inventive dunce. Despairing of getting him to accept the problem in the proper puzzler's spirit, you tel him the solution. To your further annoyance, he attacks your solution on the grounds that the boat might have a leak or lack oars. After you rectify that omission from the statement of the problem, he suggests that a see monster may swim up the river and may swallow the boat. Again you are frustrated, and you look for a mode of reasoning that will settle his hash once and for all.

McCarthy proposes *circumscription* as a technique for solving his puzzle. He argues that it is a part of common knowledge that a boat can be used to cross the river *unless there is something with it or something else prevents using it*, and if our facts do not require that there be something that prevents crossing the river, *circumscription* will generate the conjecture that there isn't. Among the various competing approaches to model the common sense reasoning *circumscription* appears to have the most advocates.

One of the serious difficulties is that the circumscription axiom itself involves a quantification over predicates, and there is therefore a sentence in *second-order logic*. Little is known about automated deduction using second-order sentences, but on the other hand Lifschits has shown in 1987 that in some special cases the circumscription is equivalent to a first order sentence. In this way we can go back, in those cases, to our secure and well known classical logic.

HOMEWORK.

1. Write definition of *logical* and *semantical* paradox.

2. Give an example of a *logical paradox.*

3. Give an example of a *semantical paradox.*

4. Describe a difference between *logical* and *semantical* paradoxes.

5. Describe a role of paradoxes in the development of classical logic and foundations of mathematics.

6. Write a definition os a *non-standard logic.*

7. Give an example of some non-standard logics.

8. Describe a difference between classical and intuitionistic logic.

9. Give two examples of Computer Science Puzzles.

10. What a *default reasoning* is? Give an example.

11. What a *non - monotonic reasoning* is? Give an example.

12. What an *auto-epistemic reasoning* is? Give an example.