CSE541   EXERCISE 1

SOLVE ALL PROBLEMS as PRACTICE and only AFTER look at the SOLUTIONS!!

Please write solutions very carefully. The grade you receive on TESTS depends not only on the fact that you SOLVE the problem, but also (30%) on elegance of your solution. Use examples from the book as a learning material of how to write solutions properly

QUESTION 1 Describe a difference between logical and semantical paradoxes.

QUESTION 2

1. Write the following natural language statement:

   From the fact that it is possible that Chris is not a boy we deduce that it is not possible that Chris is not a boy or, if it is possible that Chris is not a boy, then it is not necessary that Anne is pretty.

   as a formula

   (i) $A_1 \in \mathcal{F}_1$ of a language $\mathcal{L}_{\{\neg,\mathbf{C},\mathbf{I},\cap,\cup,\Rightarrow\}}$,

   (ii) $A_2 \in \mathcal{F}_2$ of a language $\mathcal{L}_{\{\neg,\cap,\cup,\Rightarrow\}}$.

2. Degree of the formula A_1 is: , degree of the formula A_2 is:

3. List all proper, non-atomic sub-formulas of A_1.

4. List all proper, non-atomic sub-formulas of A_2 are:

5. Find a model (restricted) and a restricted counter-model for A_2 (classical semantics). Use short-hand notation. Show work.

6. There are   possible counter-models restricted to A_2.

7. There   possible models restricted to A_2. (Don't need to list them, just justify your answer).

8. List 3 models and 3 counter-models for A_2 by extending the restricted model and the counter-model you have found in 4. to the set VAR of all variables.

9. There are   possible models for A_2.
   There are   possible counter-models for A_2.

QUESTION 3 Show that

$$\models (\neg((a \cap \neg b) \Rightarrow ((c \Rightarrow (\neg f \cup d)) \cup e)) \Rightarrow ((a \cap \neg b) \cap (\neg(c \Rightarrow (\neg f \cup d)) \cap \neg e))).$$