CSE541 EXERCISE 3

SOLVE ALL PROBLEMS as PRACTICE and only AFTER look at the SOLUTIONS!!

Reminder: We define **H** semantics operations $\cup$ and $\cap$ as follows

$$a \cup b = max\{a, b\}, \quad a \cap b = min\{a, b\}.$$

The Truth Tables for Implication and Negation are:

H-Implication

$\Rightarrow$	F	$\bot$	T
F	T	T	T
$\bot$	F	T	T
T	F	$\bot$	T

H Negation

$\neg$	F	$\bot$	T
	T	F	F

QUESTION 1 **We know** that

$$v : VAR \longrightarrow \{F, \bot, T\}$$

is such that

$$v^*((a \cap b) \Rightarrow (a \Rightarrow c)) = \bot$$

under **H** semantics. **evaluate** $v^*(((b \Rightarrow a) \Rightarrow (a \Rightarrow \neg c)) \cup (a \Rightarrow b))$.

QUESTION 2

We define a 4 valued $\mathbf{Ł}_4$ logic semantics as follows. The language is $\mathcal{L} = \mathcal{L}_{\{\neg, \Rightarrow, \cup, \cap\}}$. The logical connectives $\neg, \Rightarrow, \cup, \cap$ of $\mathbf{Ł}_4$ are operations in the set $\{F, \bot_1, \bot_2, T\}$, where $\{F < \bot_1 < \bot_2 < T\}$, defined as follows.

Negation $\neg$ is a function $\neg : \{F, \bot_1, \bot_2, T\} \longrightarrow \{F, \bot_1, \bot_2, T\}$, such that

$$\neg\bot_1 = \bot_1, \quad \neg\bot_2 = \bot_2, \quad \neg F = T, \quad \neg T = F.$$

Conjunction $\cap$ is a function $\cap : \{F, \bot_1, \bot_2, T\} \times \{F, \bot_1, \bot_2, T\} \longrightarrow \{F, \bot_1, \bot_2, , T\}$, such that for any $a, b \in \{F, \bot_1, \bot_2, T\}$, $a \cap b = min\{a, b\}$.

Disjunction $\cup$ is a function $\cup: \{F, \bot_1, \bot_2, T\} \times \{F, \bot_1, \bot_2, T\} \longrightarrow \{F, \bot_1, \bot_2, T\}$, such that for any $a, b \in \{F, \bot_1, \bot_2, T\}$, $\quad a \cup b = max\{a, b\}$.

Implication $\Rightarrow$ is a function $\Rightarrow$: $\{F, \bot_1, \bot_2, T\} \times \{F, \bot_1, \bot_2, T\} \longrightarrow \{F, \bot_1, \bot_2, T\}$, such that for any $a, b \in \{F, \bot_1, \bot_2, T\}$,

$$a \Rightarrow b = \begin{cases} \neg a \cup b & \text{if } a > b \\ T & \text{otherwise} \end{cases}$$

Part 1 Write all Truth Tables for $\mathbf{Ł}_4$

Part 2 Verify whether

$$\models_{Ł_4} ((a \Rightarrow b) \Rightarrow (\neg a \cup b))$$

QUESTION 3 Prove using proper logical equivalences (list them at each step) that

1. $\neg(A \Leftrightarrow B) \equiv ((A \cap \neg B) \cup (\neg A \cap B))$,

2. $((B \cap \neg C) \Rightarrow (\neg A \cup B)) \equiv ((B \Rightarrow C) \cup (A \Rightarrow B))$.

QUESTION 4 We define an EQUIVALENCE of LANGUAGES as follows:

Given two languages:
$\mathcal{L}_1 = \mathcal{L}_{CON_1}$ and $\mathcal{L}_2 = \mathcal{L}_{CON_2}$, for $CON_1 \neq CON_2$.
We say that they are **logically equivalent**, i.e.

$$\mathcal{L}_1 \equiv \mathcal{L}_2$$

if and only if the following conditions **C1, C2** hold.

C1: For every formula A of $\mathcal{L}_1$, there is a formula B of $\mathcal{L}_2$, such that

$$A \equiv B,$$

C2: For every formula C of $\mathcal{L}_2$, there is a formula D of $\mathcal{L}_1$, such that

$$C \equiv D.$$

Prove that $\mathcal{L}_{\{\neg, \cap\}} \equiv \mathcal{L}_{\{\neg, \Rightarrow\}}$.