CSE541 EXERCISE 4

QUESTION 1 **Use the fact** that $v : VAR \longrightarrow \{F, \perp, T\}$ be such that $v^*((a \cap b) \Rightarrow \neg b) = \perp$ under **Ł** semantics **to evaluate** $v^*(((b \Rightarrow \neg a) \Rightarrow (a \Rightarrow \neg b)) \cup (a \Rightarrow b))$. Use shorthand notation.

L semantics is defined as follows.

Ł Negation

$\neg$	F	$\perp$	T
	T	$\perp$	F

Ł Disjunction

$\cup$	F	$\perp$	T
F	F	$\perp$	T
$\perp$	$\perp$	$\perp$	T
T	T	T	T

Ł Conjunction

$\cap$	F	$\perp$	T
F	F	F	F
$\perp$	F	$\perp$	$\perp$
T	F	$\perp$	T

Ł-Implication

$\Rightarrow$	F	$\perp$	T
F	T	T	T
$\perp$	$\perp$	T	T
T	F	$\perp$	T

QUESTION 2 Prove using proper logical equivalences (list them at each step) that

$$\neg((A \Rightarrow \neg B) \cup (B \Rightarrow \neg A)) \equiv (A \cap B).$$

QUESTION 3 We define an EQUIVALENCE of LANGUAGES as follows:

Given two languages:
$\mathcal{L}_1 = \mathcal{L}_{CON_1}$ and $\mathcal{L}_2 = \mathcal{L}_{CON_2}$, for $CON_1 \neq CON_2$.
We say that they are **logically equivalent**, i.e.

$$\mathcal{L}_1 \equiv \mathcal{L}_2$$

if and only if the following conditions **C1, C2** hold.

C1: For every formula A of $\mathcal{L}_1$, there is a formula B of $\mathcal{L}_2$, such that

$$A \equiv B,$$

C2: For every formula C of $\mathcal{L}_2$, there is a formula D of $\mathcal{L}_1$, such that

$$C \equiv D.$$

Prove that $\mathcal{L}_{\{\neg, \cap, \Rightarrow\}} \equiv \mathcal{L}_{\{\uparrow\}}$.