cse547/ams547 Midterm Spring 2017
(100pts)

NAME **ID:** **ams/cs**

There are **4 Questions**, each question is **25pts**

Read carefully all of them and write solutions on next pages in spaces provided

Useful Formulas and Theorems sheet is attached

QUESTION 1 Use repertoire method to evaluate a closed formula for

$$S_n = \sum_{k=0}^{n} (-1)^k k^2$$

Part 1 Generalize the Problem to a 4 parameter recurrence and write the standard general form of closed form formula for it.

Part 2 **1.** Use functions: $R(n) = 1$, $R(n) = (-1)^n$, $R(n) = (-1)^n n$, $R(n) = (-1)^n n^2$, for all $n \in N$ to evaluate all components of the closed form formula.

2. Evaluate S_n as a particular case of the general formula.

QUESTION 2 Evaluate the sum

$$S_n = \sum_{k=0}^{n} k5^k$$

by using the following 3 methods : perturbation method, multiple sum, summation by parts Write down, in each case, which method are you using.

QUESTION 3

Part 1 Use summation by parts to evaluate $\sum_{k=0}^{n-1} \frac{H_k}{(k+1)(k+2)}$

Part 2 Evaluate

$$\sum_{0 \le k < n} k^{\underline{m}} H_k$$

QUESTION 4 Prove that

$$\lim_{n\to\infty} \frac{c^n}{n!} = 0, forc > 0$$

QUESTION 1 Use repertoire method to evaluate a closed formula for

$$S_n = \sum\nolimits_{k=0}^{n} (-1)^k k^2$$

PART 1 Generalize the Problem to a 4 parameter recurrence and write the standard general form of closed form formula for it.

The recurrence form of the summation is : $S_0 =$, $S_n =$

4 parameter recurrence is :

General form of CLOSED Formula is :

PART 2 **1.** Use functions: $R(n) = 1$, $R(n) = (-1)^n$, $R(n) = (-1)^n n$, $R(n) = (-1)^n n^2$, for all $n \in N$ to evaluate all components of the closed form formula.

2. Evaluate S_n as a particular case of the general formula.

Solution space

Solution space

QUESTION 2 Evaluate the sum $S_n = \sum_{k=0}^{n} k5^k$ by using the following 3 methods : perturbation method, multiple sum, summation by parts. Write down, in each case, which method are you using.

Solution space

QUESTION 3

Part 1 Use summation by parts to evaluate $\sum_{k=0}^{n-1} \frac{H_k}{(k+1)(k+2)}$

Part 2 Evaluate

$$\sum_{0\le k<n} k^{\underline{m}} H_k$$

QUESTION 4

Prove that

$$\lim_{n\to\infty} \frac{c^n}{n!} = 0, for c > 0$$

1 Useful Formulas

$$x^{\underline{m}} = \overbrace{x(x-1)\dots(x-m+1)}^{\text{m factors}}, \qquad integer\ m \geq 0$$

$$x^{\overline{m}} = \overbrace{x(x+1)\dots(x+m-1)}^{\text{m factors}}, \qquad integer\ m \geq 0$$

$$x^{\underline{-m}} = \frac{1}{(x+1)(x+2)\dots(x+m)}, \qquad for\ m > 0$$

$$x^{\underline{m+n}} = x^{\underline{m}}(x-m)^{\underline{n}}, \qquad integers\ m\ and\ n$$

$$\mathrm{D}f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \qquad \Delta f(x) = f(x+1) - f(x)$$

$$\mathrm{D}(x^m) = mx^{m-1}, \qquad \Delta(x^{\underline{m}}) = mx^{\underline{m-1}}$$

We define a shift operator: $\boxed{Ev(x) = v(x+1)}$

We proved $\boxed{\Delta(uv) = u\Delta v + Ev\Delta u}$

Summation by parts

$$\boxed{\sum u\,\delta v = uv - \sum Ev\delta u}$$

USEFUL THEOREMS

THEOREM 1

If the infinite sum

$$\Sigma_{n=1}^{\infty} a_n \text{ converges, then } \lim_{n \to \infty} a_n = 0.$$

THEOREM 2 (D'Alambert's Criterium)

Any series with all its terms being positive real numbers, i.e.

$$\Sigma_{n=1}^{\infty} a_n, \text{ for } a_n \geq 0, a_n \in R$$

converges if the following condition holds:

$$\lim_{n \to \infty} \frac{a_n}{a_{n+1}} < 1.$$