CHAPTER 5 EXERCISE # 4

### THE PROBLEM

**EVALUATE:** 

# $\begin{pmatrix} -1 \\ k \end{pmatrix}$ by negating its upper index.

(k in Z)

## What is Upper Index Negation ?

The following identity:

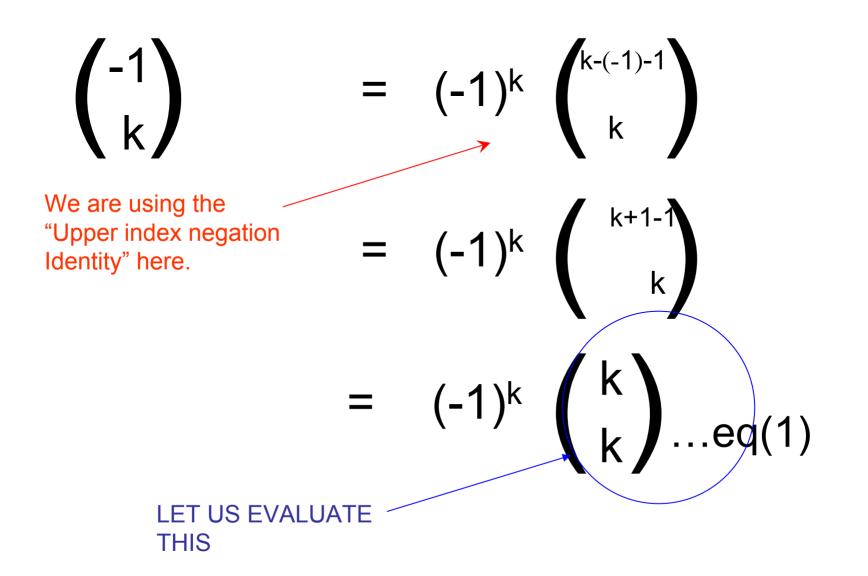
# $\begin{pmatrix} r \\ k \end{pmatrix} = (-1)^k \begin{pmatrix} k-r-1 \\ k \end{pmatrix}$

# (where k in ${f Z}$ and r in ${f R}$ )

is called the upper index negation identity.

(Proof of this identity is given in lecture notesslide 40 of chapter 5; and pg 164 of the book.)

### SOLUTION TO THE PROBLEM



# EVALUATING SUB-PROBLEM In order to be able to solve $\begin{pmatrix} k \\ k \end{pmatrix}$ , we need

to first look at the following formal definition, "def1":

$$\begin{pmatrix} r \\ z \end{pmatrix} = 0 , \text{ if } z < 0.$$

$$\begin{pmatrix} r \\ z \end{pmatrix} = \frac{r^{\underline{k}}}{k!} , \text{ if } z \ge 0.$$

$$Z \text{ in } \overline{R}$$

### SOLUTION TO THE SUB-PROBLEM

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Hence in order to be able to solve, **K** 

(where k ranges over all integers), it is obvious from the definition above that we need to consider two cases. One case when k < 0, and another case when  $k \ge 0$ .

### SOLUTION TO THE SUB-PROBLEM

• Case When k <0, (then by def 1):

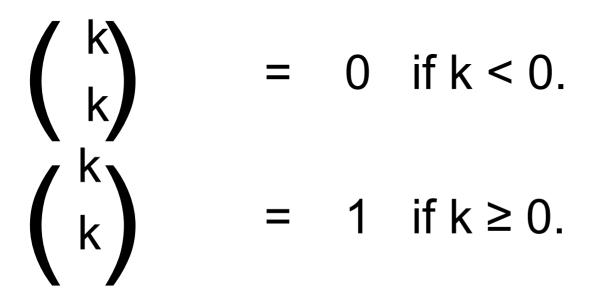
k  $= \frac{k^{\underline{k}}}{k!} = \frac{k(k-1)(k-2)\dots(k-k+1)}{k!}$ • Case When  $k \ge 0$ , (then by def 1): k

### GOING BACK TO THE PROBLEM

• Now, we had proven before that:

$$\begin{pmatrix} -1 \\ k \end{pmatrix} = (-1)^k \begin{pmatrix} k \\ k \end{pmatrix}$$

And we have just proven that:



# SOLUTION TO THE PROBLEM

• Solution When k<0:

$$\begin{pmatrix} -1 \\ k \end{pmatrix} = (-1)^{k} \begin{pmatrix} k \\ k \end{pmatrix} = (-1)^{k} (0) = 0$$

• Solution When  $k \ge 0$ :

$$\begin{pmatrix} -1 \\ k \end{pmatrix} = (-1)^{k} \begin{pmatrix} k \\ k \end{pmatrix}$$
$$= (-1)^{k} (1) = (-1)^{k}$$

### FINAL ANSWER

• When k<0:

$$\begin{pmatrix} -1 \\ k \end{pmatrix} = 0$$

• When  $k \ge 0$ :

$$\begin{pmatrix} -1 \\ k \end{pmatrix} = (-1)^{k}$$