NUMBER THEORY

BOOK CHAPTER 4

+ Number Theory and its History

by

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(1899 – 1968)

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NUMBERS AND COUNTING

According to anthropologists, every people has some terminology for the first numbers.
In some most primitive tribes this may not extend beyond TWO or THREE.

BASIC NUMBER GROUPS

BASIC : DECIMAL OR DECADIC
group of 10 objects
OFTEN in the WORD for 10 signifies ONE MAN!

QUINARY SYSTEMS : based on groups of 5 (or ONE HAND)

VIGESIMAL SYSTEMS : based on groups of 20 (hands and toes)

VIDELY used between AMERICAN INDIAN
most well-developed was MAYAN system

QUATRE-VINGT for 80 in French is a remnant of old 20-system; also in Denmark!

BABYLONIAN SEXAGESIMAL SYSTEM

- the largest known basic number 60

difficult to explain the reason!

but we use it when measuring TIME

and ANGLES.

DUODECIMAL SYSTEM - base 12

we still count in DOZENS and gross

CERTAIN AFRICAN tribes use basis 3 or 4

BINARY or DYADIC SYSTEM - base 2

has been used by Australian indigenes

Traces in linguistic

ELEVEN - one left over (ten and one)

TWELVE - two over

TEN - may be derived from an Indo-European root meaning TWO HANDS

HUNDRED - from ten times (ten)

Names for THOUSAND are unrelated in different branches of Indo-European; so it might be much later construction.

NUMBER SYSTEM with BASE b

is a system in which we represent the numbers in the form

$$a_n b^n + \dots \; a_2 b^2 + a_1 b + a_0$$

where the coefficients a_i are numbers from 0 to $b-1$ i.e

$$0 \le a_i \le b-1$$

The MAYAN number system was develop to unusually high levels, but the system has one peculiar irregularity: **BASIC group is 20**, but the group of second order is **NOT $20 \times 20 = 400$**, but **$20 \times 18 = 360$**!

This appears to be connected with the DIVISION of MAYAN year into 18 months, suplemented with 5 extra days.

the higher groups are

360×20, **360×20^2** ... etc

WRITING NUMBERS

EARLY EGYPTIAN NUMERALS 3400 BC

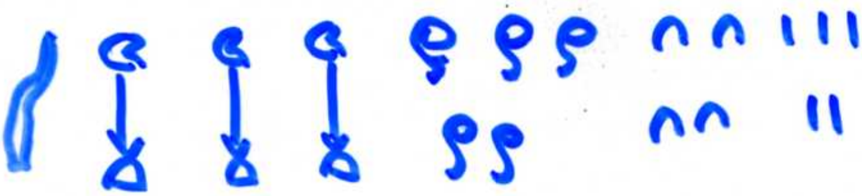
(Hieroglyphic)

1 2 3 9 10 100

1000 10,000

SIMPLE GROUPING SYSTEMS

EXAMPLE



= 13,545

ROMAN NUMERALS

1	2	5	10	50	100	500	1,000
I	II	V	X	L	C	D	M

EXAMPLE — **SIMPLE GROUPING:**

MDCCCXXVII = 1,827

Substruction principle: a smaller unit preceding a higher one indicates **SUBSTRUCTION**

Example

IX = 9   IV = 4

Another examples of a simple grouping systems follow.

ATTIC or HERODIANIC GREEK NUMERALS

1	5	10	100	1000	10,000
Ι	Γ	Δ	Η	Χ	Μ

EXAMPLE

Χ 𐅅 Η Η Η Δ Δ Γ Ι Ι = 1827

SYMBOLS are derived from the initials of the Greek numbers:

ΠΕΝΤΕ (5) ΔΕΚΑ (10)

ΗΚΑΤΟΝ (100) ΧΙΛΙΟΣ (1,000)

ΜΥΡΙΟΣ (10,000)

MULTIPLICATIVE GROUPING SYSTEMS

The simple grouping system in several instances into a type of numeration that has special CIPHERS for the numbers in the BASIC GROUP e.g 1, ... 9 and a SECOND CLASS of symbols for the higher groups e.g.

10 = t , 100 = h , 1,000 = th

The CIPHERS are then used MULTIPLICATIVERY to show how many of the higher group should be indicated

EXAMPLE

3,297 = 3th 2h 9t 7

TRADITIONAL CHINESE - JAPANESE

SYSTEM is a MULTIPLICATIVE

CHINESE - JAPANESE NUMERALS

1	一	10	十
2	二	100	百
3	三	1000	千
4	四		
5	五		
6	六		
7	七		
8	八		
9	九		

EXAMPLE

3468

三
千
四
百
六
十
八

SIMPLIFIED
everyday
life

CHINESE MERCANTILE
1-10

〡 〢 〣 〤 〥 〦 〧 〨 〩 十

3468

叁仟肆佰陆拾捌

Simplified

upper case

參仟肆佰陸拾捌

TRADITIONAL CHINESE

upper case

used as currency

Everyday simplified is lower case (arabic form)

CIPHERED NUMERAL SYSTEM

We use symbols for number from 1 to 9; multiples of 10 to 90; hundreds up to 900; and so on.

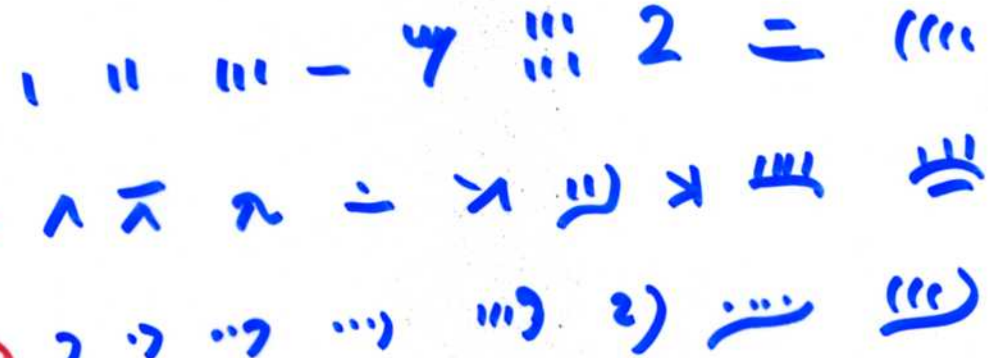
All numbers are represented as a combination of such symbols

HIERATIC NUMERAL (Egyptian heliographic)

1-9

10-90

10-900



The Greek CIPHERED – the letter of alphabet + symbols from Semitic

ALPHABETIC GREEK NUMERALS

1-9: α β γ δ ε ς ζ η θ

10-90: ι κ λ μ ν ξ ο π ϙ

100-900: ρ σ τ υ φ χ ψ ω ϡ

EXAMPLE: ψμβ = 742

,α = 1,000 ,β = 2,000 etc

POSITIONAL NUMERAL SYSTEMS

* Expresses every number by means of 0, 1, 2 ... 9

PRINCIPLE of LOCAL VALUE, so that a symbol designates a value, or class which depends on the PLACE it takes in the numeral representation.

352 325 253

local 2 signifies :

2 2×10 2×100

NEED OF a symbol for ZERO !

to represent a missing, or void class

204 ≠ 24

Advantages : ① compact, ② easy readable

AND ③ possible to express arbitrary large number ~~with~~ only by the digits of BASIC GROUP

AND ④ calculations extremely simple:

HINDU-ARABIC NUMERALS

Our numerals are commonly known as **HINDU-ARABIC**

Most historical evidence points to **INDIA** as the county of their orgin. To the ARABS who were instrumental in their transmission to Europe, they were known as "Hindu numbers"

BRAHMI SYMBOLS (100 BC)

A.D 600-800 the use of POSITIONAL system with zero made appearance in INDIA

by AD 800 the system was known between Arabs in Bagdad and superseded the older type Arabic numerals.

MOHAMED IBN MUSA al KHOWARIZMI

- one of the greatest Arab mathematicians (around AD 800) contributed to the spread of calculations with the new system

work: **AL-JABR**

has given rise to the term **ALGEBRA** of modern mathematics.

Translated to **LATIN** and art of computing with Hindu-Arabic numerals became known as **ALGORISM**

The Hindus denoted **ZERO** by a dot (·) or circle (O) and used term **sūnya**, or **void** for it. Translated to arabic this became **as-sifr**

a common root for words **zero** and **cipher**.

GOBAR or WESTERN ARABIC NUMERALS (1000 AD)

1 2 3 4 5 6 7 8 9 0

Appeared in SPAIN.

The name GOBAR or DUST numerals is derived from the Indian custom of calculating on the ground, or board covered with sand.

The earlies manuscript using Gobar numerals dates AD 976

11-12 century Europeans went to SPAIN to learn ARAB learning

DEVELOPEMENT

AD 1202

LIBER ABACI, a compedium of arithmetic, algebra and number theory by **LEONARDO FIBONACCI** (or PISANO)

1250

ALGORISMUS of John of Halifax

1543 **Nicolaus Copernicus**

De revolutionibus orbium coelestium

uses mixture of Roman and Hindu-Arabic numerals and numbers written in plain words

1585 - decimal fractions

SIMON STEVIN **LA DISME**

John Napier was first to use a common point to separate decimals from integers as we do.