CSE 548 Homework Assignment 5

Due in Class on Thursday, December 1

November 18, 2009

1 (Textbook *22.2-7*)
There are two types of professional wrestlers: "babyfaces" ("good guys") and "heels" ("bad guys"). Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have n professional wrestlers and we have a list of r pairs of wrestlers for which there are rivalries. Give an $O(n + r)$-time algorithm that determines whether it is possible to designate some of the wrestlers as babyfaces and the remainder as heels such that each rivalry is between a babyface and a heel. If it is possible to perform such a designation, your algorithm should produce it.

2 (Textbook *22.3-13*, modified)
A directed graph $G = (V, E)$ is *singly connected* if $u \rightarrow v$ implies that G contains at most one simple path from u to v for all vertices $u, v \in V$. Explain how you can use DFS to determine whether or not a directed graph is singly connected. Justify your answer.

3 (Textbook *22.4-3*, modified)
Describe how you can give an algorithm that determines whether or not a given undirected graph $G = (V, E)$ contains a cycle. Your algorithm should run in $O(V)$ time, independent of $|E|$. (Hint: If G is a connected acyclic graph, $|E| = |V| - 1$ (Textbook p.1174).)

4 (Textbook *22-3 Euler tour*)
An *Euler tour* of a strongly connected, directed graph $G = (V, E)$ is a cycle that traverses each edge of G exactly once, although it may visit a vertex more than once.

a. Show that G has an Euler tour if and only if in-degree(v) = out-degree(v) for each vertex $v \in V$.

b. Describe an $O(E)$-time algorithm to find an Euler tour of G if one exists. (Hint: Merge edge-disjoint cycles.)