1 Queueing with vacation – a Markov Chain

1.1 State transition diagram

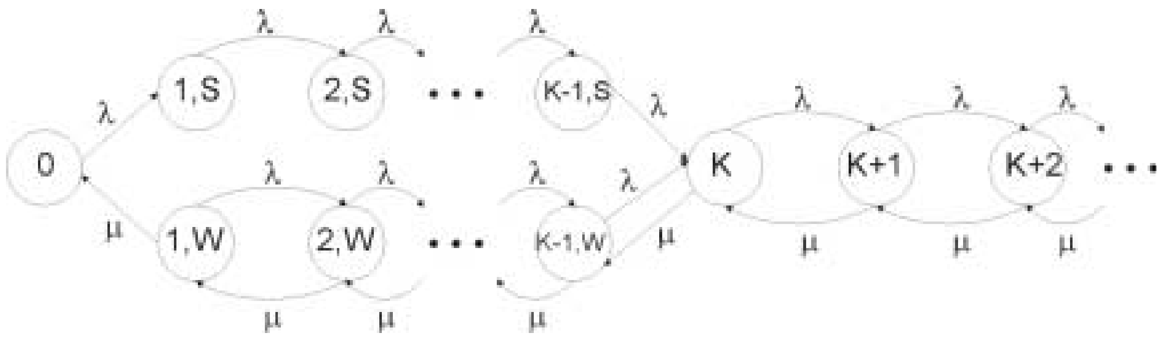

Figure 2.1 State transition diagram for the Markov Chain

1.2 Steady state balance equations

$$\left.\text{state } 0: \underbrace{\pi_{1,W} \cdot \mu}_{\text{in}} = \underbrace{\pi_0 \cdot \lambda}_{out}\right\} \text{type } 0$$

$$\left.\begin{array}{l}\text{state } 1,\text{S}: \pi_0 \cdot \lambda = \pi_{1,S} \cdot \lambda \\ \text{state } 2,\text{S}: \pi_{1,S} \cdot \lambda = \pi_{2,S} \cdot \lambda \\ \vdots \\ \text{state k - 2,S}: \pi_{k-3,S} \cdot \lambda = \pi_{k-2,S} \cdot \lambda \\ \text{state k -1,S}: \pi_{k-2,S} \cdot \lambda = \pi_{k-1,S} \cdot \lambda \end{array}\right\} \text{type n,S}, \ (1 \le n \le k-1)$$

$$\left.\begin{array}{l}\text{state } 1,\text{W}: \pi_{2,W} \cdot \mu = \pi_{1,W} \cdot (\lambda+\mu) \\ \text{state } 2,\text{W}: \pi_{1,W} \cdot \lambda + \pi_{3,W} \cdot \mu = \pi_{2,W} \cdot (\lambda+\mu) \\ \vdots \\ \text{state k - 2, W}: \pi_{k-3,W} \cdot \lambda + \pi_{k-1,W} \cdot \mu = \pi_{k-2,W} \cdot (\lambda+\mu) \\ \text{state k -1, W}: \pi_{k-2,W} \cdot \lambda + \pi_{k} \cdot \mu = \pi_{k-1,W} \cdot (\lambda+\mu) \end{array}\right\} \text{type n,W}, \ (1 \le n \le k-1)$$

$$\left.\begin{array}{l}\text{state k}: (\pi_{k-1,W} + \pi_{k-1,S}) \cdot \lambda + \pi_{k+1} \cdot \mu = \pi_k \cdot (\lambda + \mu) \\ \text{state k}+1: \pi_k \cdot \lambda + \pi_{k+2} \cdot \mu = \pi_{k+1} \cdot (\lambda + \mu) \\ \vdots \\ \text{state } \underset{(i \geq i)}{\text{k}} + \text{i}: \pi_{k+i-1} \cdot \lambda + \pi_{k+i+1} \cdot \mu = \pi_{k+i} \cdot (\lambda + \mu)\end{array}\right\} \text{type n, } (\text{n} \geq \text{k})$$

1.3 Steady state probabilities of sleeping states

All steady state propabilities, $\pi_{n,S}, 1 \leq n \leq k-1$, are equal to π_0 as shown below.

$$\text{state } 1,\text{S}: \pi_0 \cdot \lambda = \pi_{1,S} \cdot \lambda \Rightarrow \pi_{1,S} = \pi_0 \cdot \frac{\lambda}{\lambda} = \pi_0$$

$$\text{state } 2,\text{S}: \pi_{1,S} \cdot \lambda = \pi_{2,S} \cdot \lambda \Rightarrow \pi_{2,S} = \pi_{1,S} \cdot \frac{\lambda}{\lambda} = \pi_0$$

$$\vdots$$

$$\text{state k - }2,\text{S}: \pi_{k-3,S} \cdot \lambda = \pi_{k-2,S} \cdot \lambda \Rightarrow \pi_{k-2,S} = \pi_{k-3,S} \cdot \frac{\lambda}{\lambda} = \pi_0$$

$$\text{state k - }1,\text{S}: \pi_{k-2,S} \cdot \lambda = \pi_{k-1,S} \cdot \lambda \Rightarrow \pi_{k-1,S} = \pi_{k-2,S} \cdot \frac{\lambda}{\lambda} = \pi_0$$

1.4 Steady state probability for state 2,W

$$\text{state } 0: \pi_{1,W} \cdot \mu = \pi_0 \cdot \lambda \Rightarrow \pi_{1,W} = \pi_0 \cdot \frac{\lambda}{\mu}$$

$$\text{state } 1,\text{W}: \pi_{2,W} \cdot \mu = \pi_{1,W} \cdot (\lambda + \mu) \Rightarrow$$

$$\pi_{2,W} = \frac{\pi_{1,W} \cdot (\lambda + \mu)}{\mu} = \frac{\pi_0 \cdot \frac{\lambda}{\mu} \cdot (\lambda + \mu)}{\mu} = \pi_0 \cdot \frac{\lambda}{\mu^2} \cdot (\lambda + \mu) = \pi_0 \cdot \left(\frac{\lambda^2}{\mu^2} + \frac{\lambda}{\mu} \right)$$

$$\Theta\ \pi_{2,W} = \pi_0 \cdot \left(\frac{\lambda^2}{\mu^2} + \frac{\lambda}{\mu} \right)$$