Halfedge Mesh Representation

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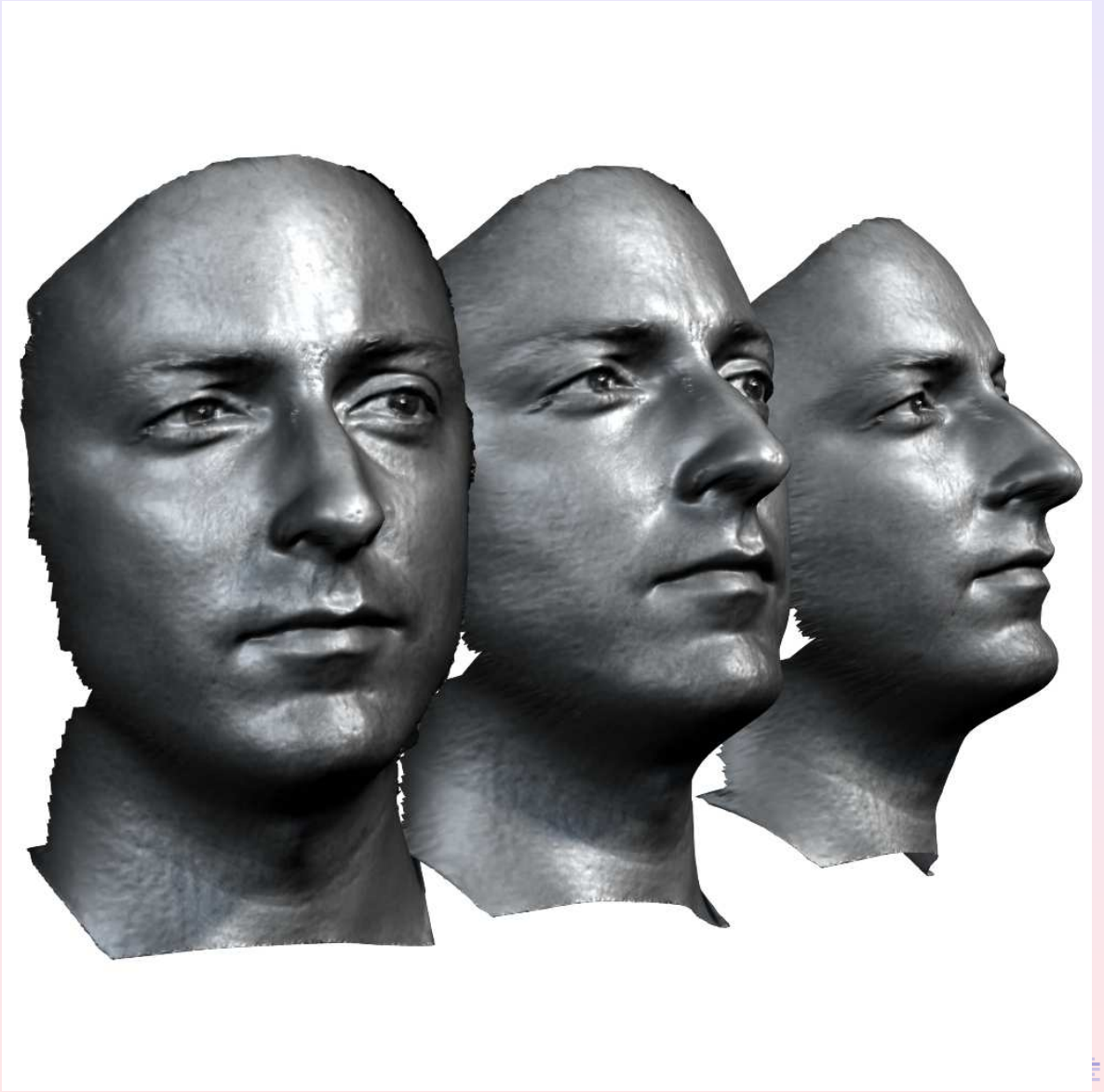
Halfedge Data Structure

Discrete Surfaces

Acquired using 3D scanner.

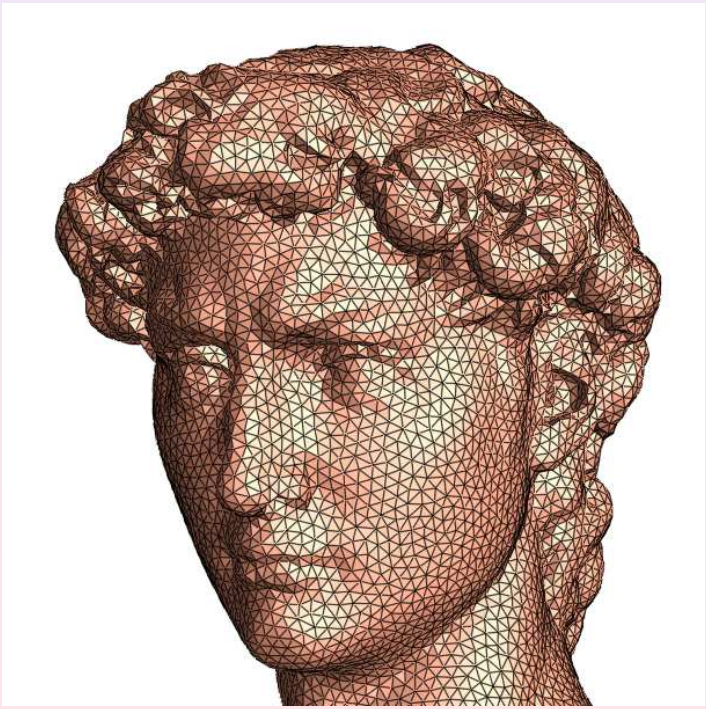


Discrete Surfaces

Our group has developed high speed 3D scanner, which can capture dynamic surfaces 180 frames per second.

Generic Surface Model - Triangular Mesh

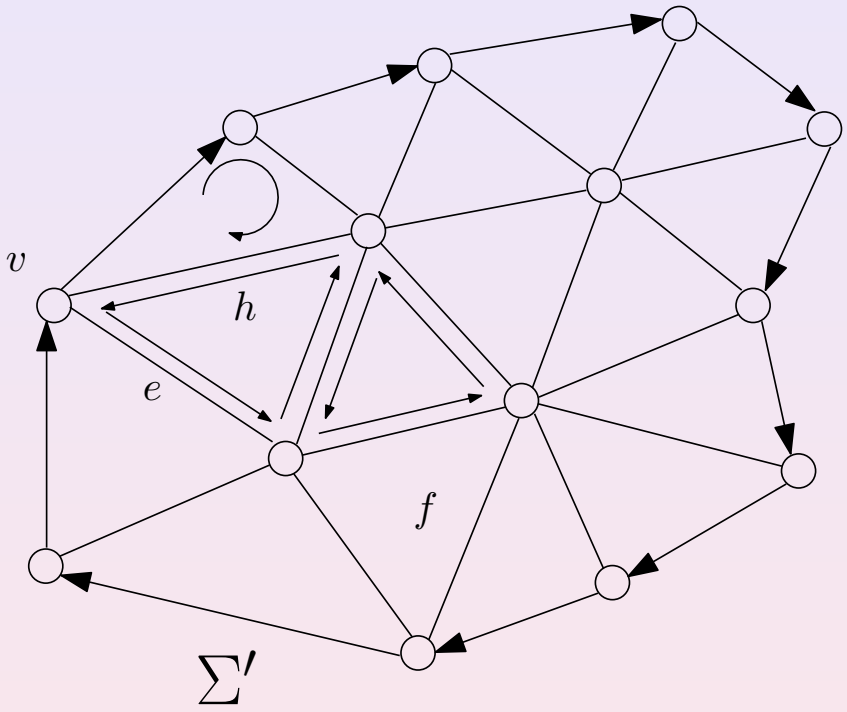
- Surfaces are represented as polyhedron triangular meshes.
- Isometric gluing of triangles in $\mathbb{E}^2$.
- Isometric gluing of triangles in $\mathbb{H}^2, \mathbb{S}^2$.



Discrete structures

- Topology - Simplicial Complex , combinatorics
- Conformal Structure - Corner angles (and other variant definitions)
- Riemannian metrics - Edge lengths
- Embedding - Vertex coordinates

Generic Surface Model - Triangular Mesh



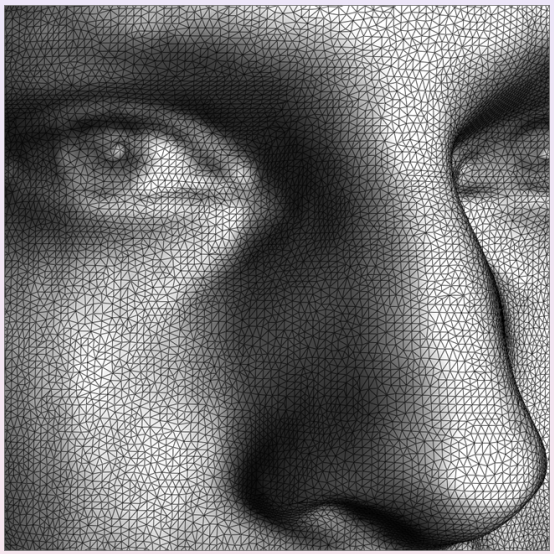
Triangle mesh

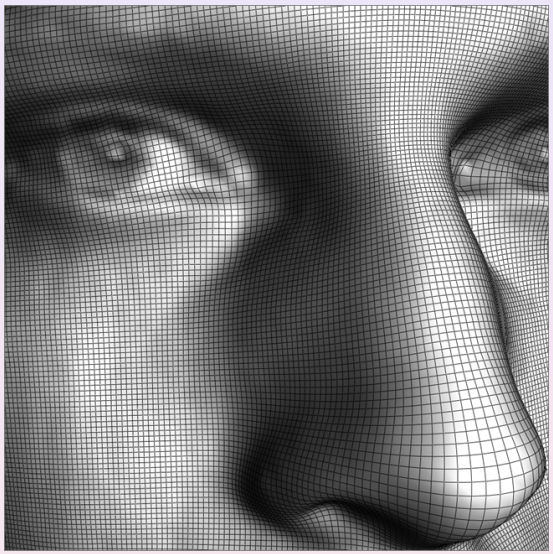
Definition (Mesh)

A triangle mesh is a oriented two dimensional simplical complex, generally embedded in $\mathbb{R}^3$.

Our goal is to design a data structure to efficiently represent general meshes.

Generic Surface Model - Triangular Mesh





halfedge data structure

fundamental classes

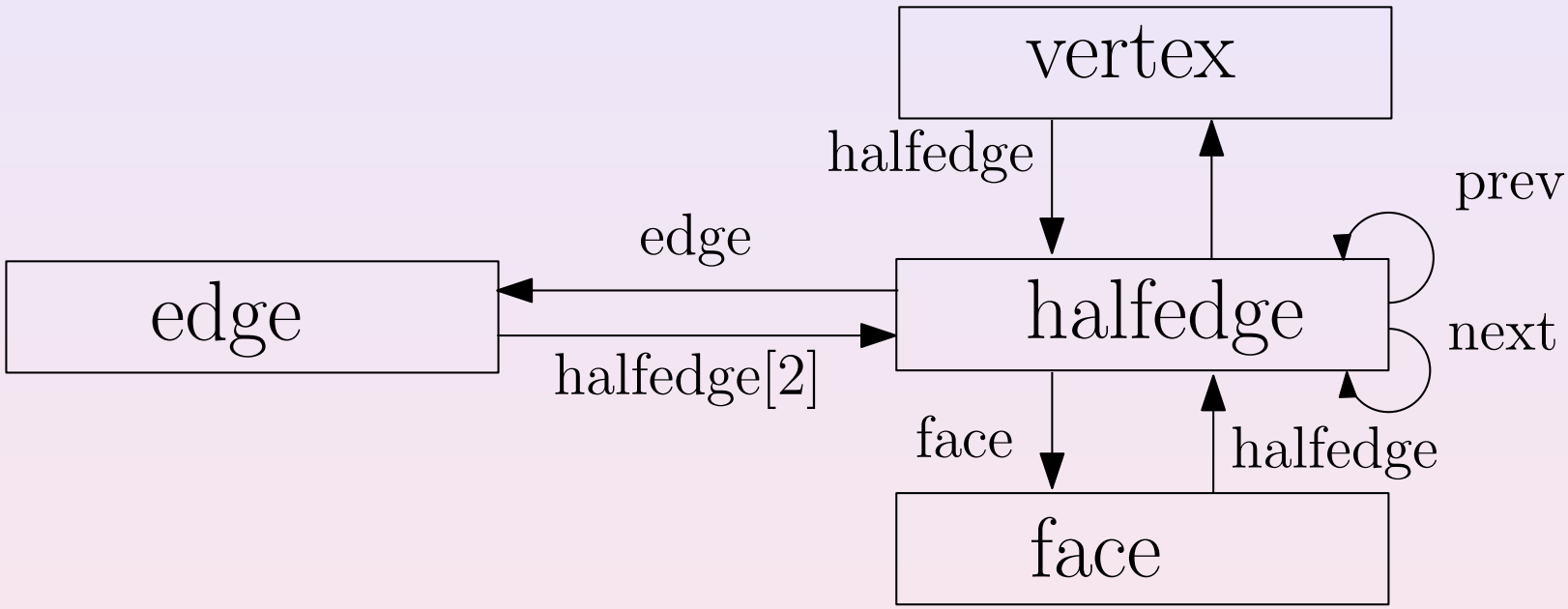
- Vertex
- Halfedge, oriented edge
- Edge, non-oriented edge
- Face, oriented

Links

All objects are linked together through pointers, such that

1. The local Eucler operation can be easily performed
2. The memory cost is minimized

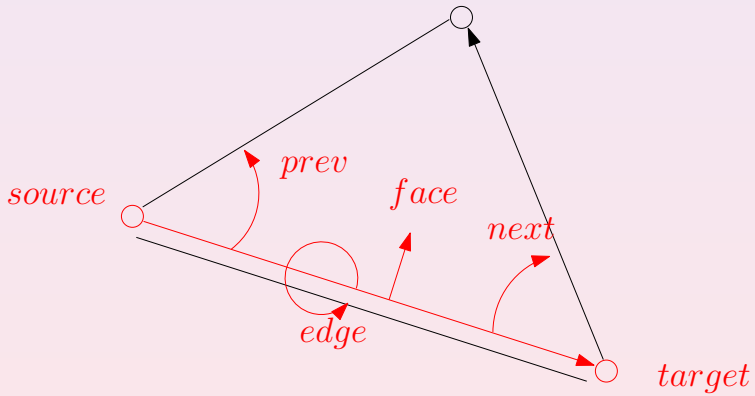
Generic Surface Model - Triangular Mesh



Halfedge class

Pointers

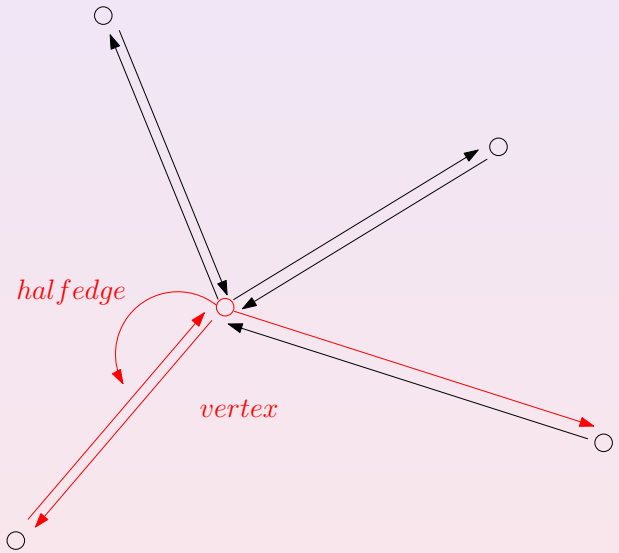
- Halfedge pointers: prev, next halfedge;
- Vertex pointers: target vertex, source vertex;
- Edge pointer: the adjacent edge;
- face pointer: the face it belongs to;



Vertex class

Pointers

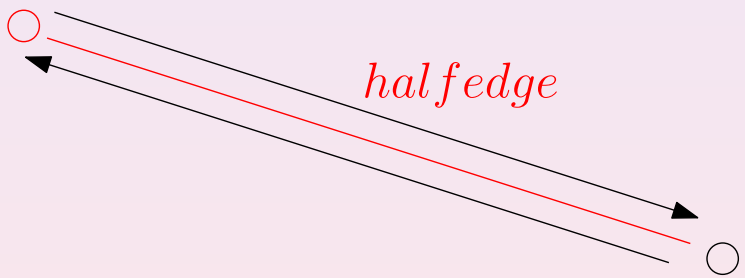
- Halfedge pointers: the first in halfedge



Edge class

Pointers

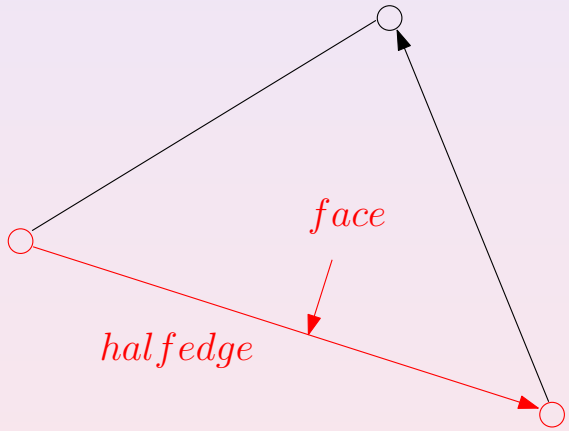
- Halfedge pointers: to the adjacent two halfedges.
- if the edge is on the boundary, then the second halfedge pointer is null.



Face class

Pointers

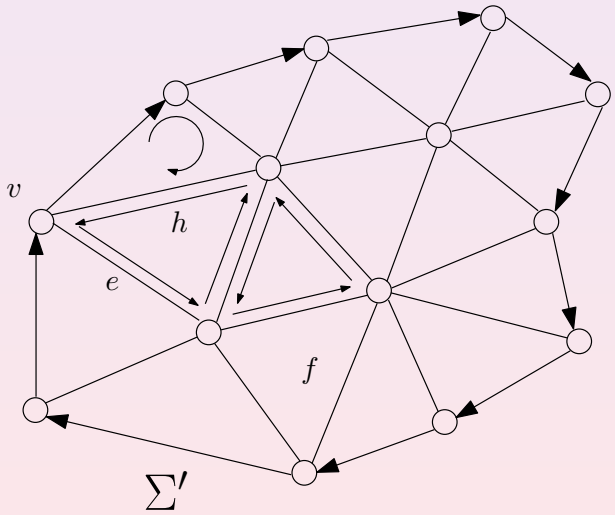
- Halfedge pointers: to the first halfedge.



Mesh class

Data members

- A list of vertices;
- A list of halfedges;
- A list of edges;
- A list of faces;



Euler Operation

circulating neighbors of a vertex $v \to v/e/f/h$

- iterate out-halfedges counter-clock-wisely
- iterate in-halfedges counter-clock-wisely
- iterate neighboring faces CCWly
- iterate neighboring vertices CCWly

Rotate a halfedge about its target vertex clwly:

$$he = he \to next() \to dual();$$

Rotate a halfedge about its target vertex ccwly:

$$he = he \to dual() \to prev();$$

Euler Operation

circulating neighbors of a face $f \rightarrow v/e/f/h$

- iterate halfedges ccwly
- iterate edges ccwly
- iterate vertices ccwly
- iterate faces ccwly

Circulate halfedges of a face ccwly:

$$he = he \rightarrow next()$$

circulate halfedge of a face clwly:

$$he = he \rightarrow prev();$$

Attributes

Each object stores attributes (traits) which defines other structures on the mesh:

- metric structure: edge length
- angle structure: halfedge
- curvature : vertex
- conformal factor: vertex
- Laplace-Beltrami operator: edge
- Ricci flow edge weight; edge
- holomorphic 1-form: halfedge