Brain Surface Parameterization with Holomorphic Differential Forms

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Objective: It is often difficult to parameterize a complex 3D surface with a mapping to a sphere or 2D plane without substantial angular or area distortion.
We present a new method to parameterize brain surfaces using holomorphic differential forms. By contrast with variational approaches based on surface ination, our method can parameterize surfaces with arbitrary complexity including branching surfaces not topologically homeomorphic to a sphere (higher-genus objects).

Methods: All orientable surfaces are Riemann surfaces and admit a global conformal structure. For high genus surfaces, a process known as holomorphic flow segmentation can be used to induce a canonical partition of the 3D surface consisting of subregions that cover the surface. Each of the subregions can be conformally mapped to a rectangle in the parameter domain.

Suppose S_1 is a regular surface. For higher genus surfaces, the local conformal parameterization can be extended to cover the whole surface except at several points (zero points). By the Riemann-Roch theorem, there are $2g - 2$ zero points on a global conformal structure of a genus g closed surface. By the Circle-Valued Morse theorem, the iso-parametric curves through the zero points segment the whole surface into a set of patches, where each patch is either a topological disk, or a cylinder. The holomorphic flow segmentation is completely determined by the surface geometry and the choice of the global conformal parameterization. The resulting segmentation and parameterization are intrinsic and stable, and are continuous across the segment boundaries in the parameterization space.

Results: We tested our algorithm for creating surface parameterizations of the hippocampus, lateral ventricles and cerebral cortex. Since the shape of hippocampus and lateral ventricles are not close to a sphere, the conventional method of mapping them to a sphere would require large distortions in some surface regions. Our method minimizes the distortion.

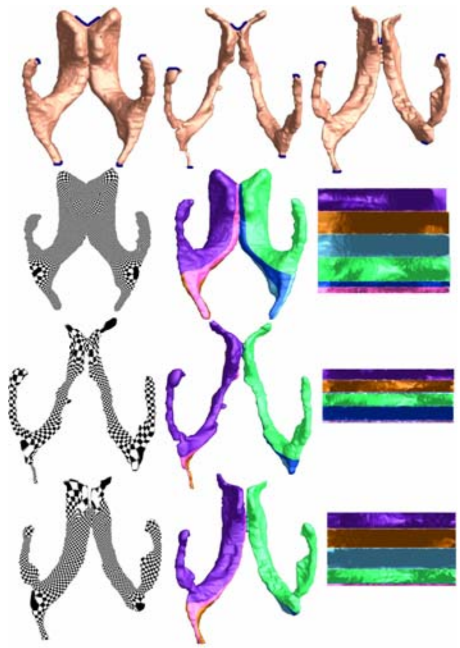

Figure 1 (left) Shows brain lateral ventricle models parameterized using holomorphic 1-forms, for a 65-year-old subject with HIV/AIDS (second row), a healthy 21-year-old subject (third row) and a second healthy 28-year-old subject (fourth row), respectively. The computed holomorphic flow segmentations and their associated sets of rectangular parameter domains are shown (the texture mapped into the parameter domain here simply corresponds to the intensity of the surface rendering, which is based on the surface normals).

Figure 2 (below) Illustrates the parameterization of cortical surfaces using the holomorphic 1-form approach. The thick lines are landmark curves, including several major sulci lying in the cortical surface. These sulcal curves are always mapped to a boundary in the parameter space images.