Holoimage

Xianfeng (David) Gu[1] Song Zhang[2] Liangjun Zhang[3]
Peisen Huang[1] Ralph Martin[4] Shing-Tung Yau[2]

[1]Engineering School
Stony Brook University

[2]Mathematics Department
Harvard University

[3]Computer Science Department
North Carolina University

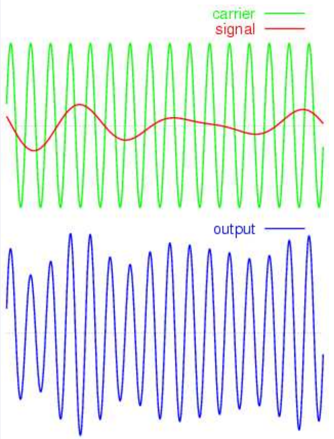
[4]Computer Science Department
Cardiff University

ACM Solid and Physical Modeling Symposium, 2006

Radio Modulation

Carrier wave $c(t) = C\sin(\omega_c t + \phi_c)$, Signal $m(t)$

Modulation



Amplitude Modulation

Add signal to the amplitude,

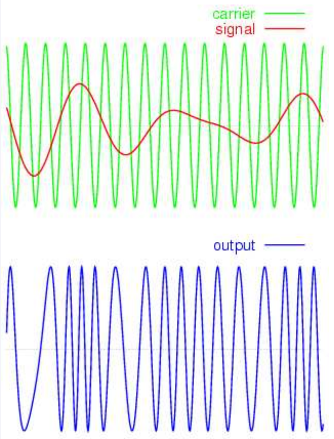
$$y(t) = (C + m(t))\sin(\omega_c t + \phi_c).$$

By demodulation, signal can be extracted from the modulated wave.

Radio Modulation

Carrier wave $c(t) = C\sin(\omega_c t + \phi_c)$, Signal $m(t)$

Modulation



Phase Modulation

Add signal to the phase,

$$y(t) = C\sin(\omega_c t + \phi_c + m(t)).$$

By demodulation, signal can be extracted from the modulated wave.

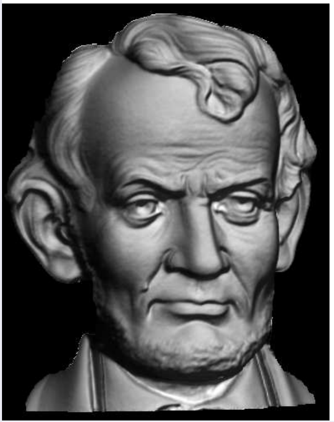
Key Idea

Modulate geometry and shading with spacial waves.

- Signal is both the geometry and the shading of a smooth surface.
- Carrier wave is the spacial carrier wave.
- Shading is encoded by amplitude modulation.
- Geometry is encoded by phase modulation.

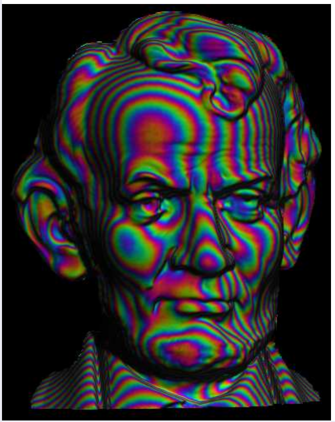
Holoimage: Represent both shading and geometry by a single image.

Holoimage



Holoimage

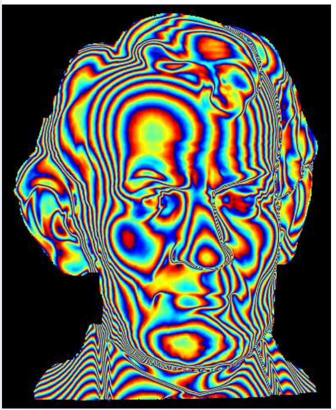
A conventional image only records amplitude information.



Holoimage

A holoimage records both amplitude and phase information.

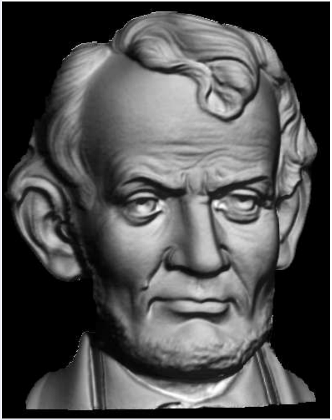
Holoimage



Holoimage

Phase map.

Holoimage

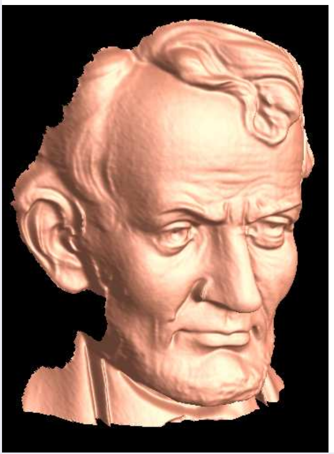


Holoimage

Intensity Map,

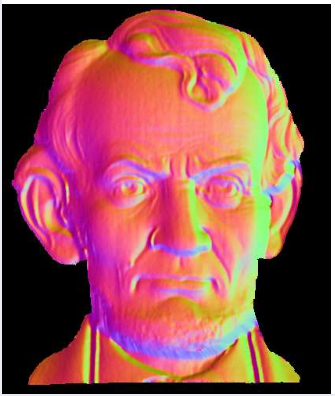
$$I(\mathbf{r}) = |a(\mathbf{r})|^2.$$

Holoimage



Holoimage

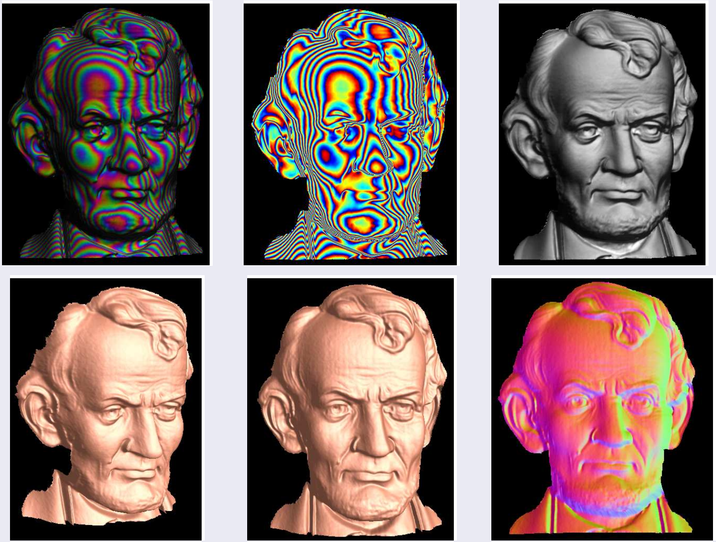
Geometry can be deduced from the phase map.



Holoimage

Normal field can be deduced from the geometry.

Holoimage



Comparison to Previous Works

- Comparison to Geometry image
  - Holoimage encodes both geometry and shading;
  - Holoimage can be captured from real life in real time;
  - Holoimage requires less bits for each pixel;
  - Holoimage can not represent the whole surface.
- Comparison to other geometric data acquisition methods:
  - Simple devices with higher acquisition speed;
  - Holoimage uses two wave length phase unwrapping, it is much simpler and can be implemented on GPU;
  - Difficult for glossy or dark surfaces.

Wave Equation

Light is a eletromagnetic field. Let $\mathbf{r} = (x, y, z)$ represent a point in the space, t represent the time, $u(\mathbf{r}, t)$ is the electric field instensity at the point $\mathbf{r}$ and time t, then wave equation is

$$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0.$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

c is the light speed. Wave equation is linear.

Wave Optics

Wave

The wave function of a monochromatic wave:

$$u(\mathbf{r}, t) = a(\mathbf{r}) \cos[2\pi\nu t + \phi(\mathbf{r})],$$

- Amplitude, *intensity*
- Frequency, *color*
- Phase, *geometry*

Light Interference

Interference

Complex wave function

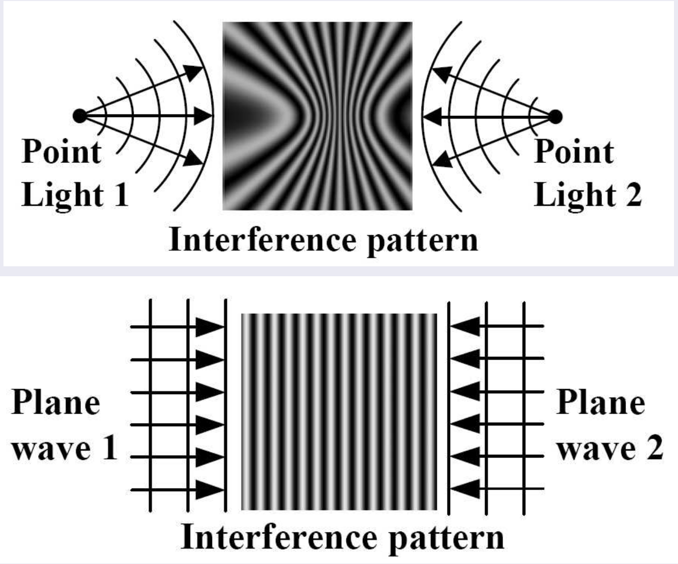
$$\Psi(\mathbf{r}, t) = a(\mathbf{r})e^{i\phi(\mathbf{r})}e^{i2\pi\nu t}.$$

Interference wave function

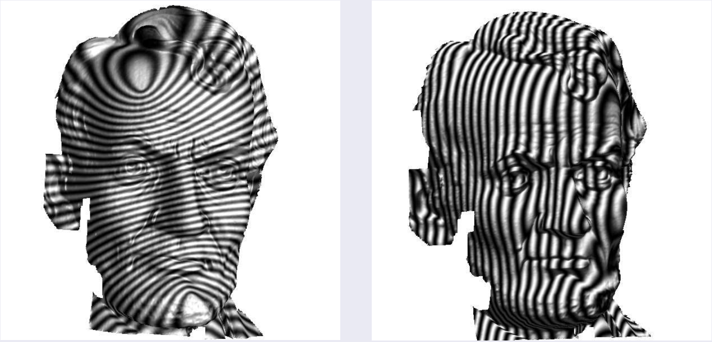
$$\Psi(\mathbf{r}, t) = \Psi_1(\mathbf{r}, t) + \Psi_2(\mathbf{r}, t)$$

intensity $I(\mathbf{r}, t) = |\Psi(\mathbf{r}, t)|^2$.

Light Interference

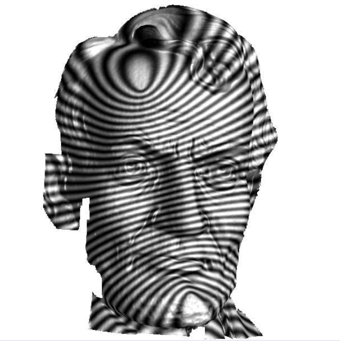


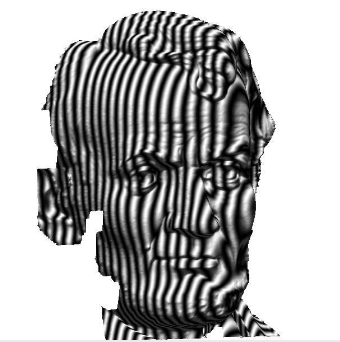
Light Interference



Idea The distortion of the fringe pattern conveys the geometric information of the surface.

Light Interference

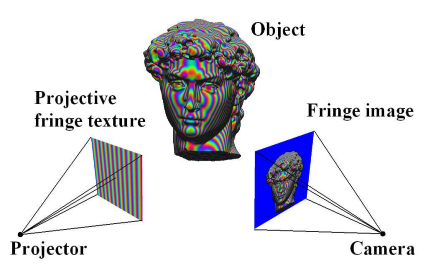




Idea The distortion of the fringe pattern conveys the geometric information of the surface.

System Set up

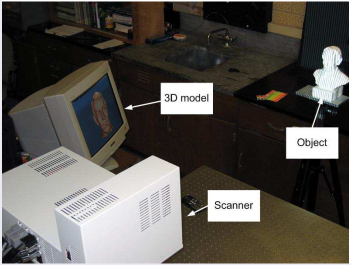
Set up



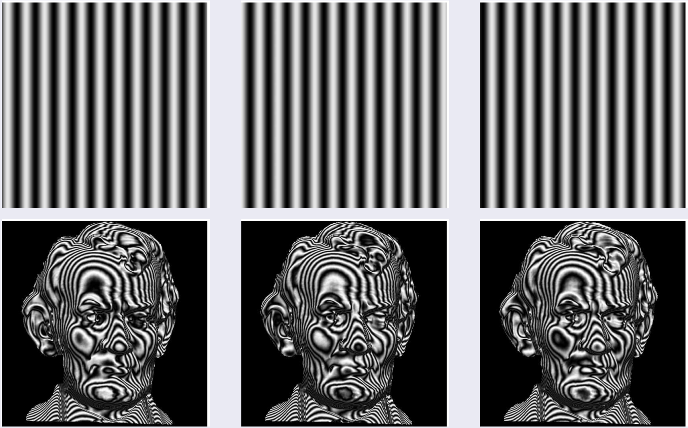
- A digital projector and a camera.
- Project sinusoidal fringe pattern.

System Set up

Projector, Camera



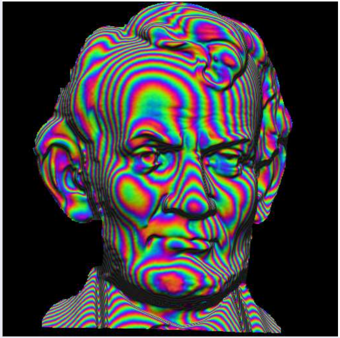
Digital Fringe Pattern



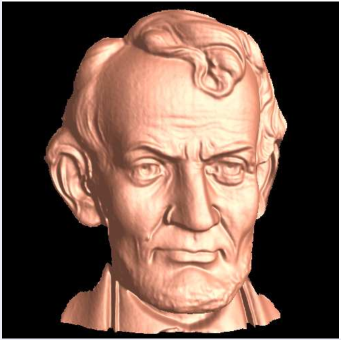
red 0° | green 120° | blue 240°

$$T_k(u,v) = 0.5 + 0.5\cos(2\pi u/\lambda + \phi_k), \phi_k = 2\pi\frac{k-1}{3}, k = 0, 1, 2.$$

Holoimage Synthesis



Holoimage



Shading Geometry

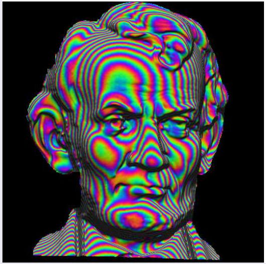
Shading Model

Shading model:

$$I_k(x,y) = a(x,y) + r(x,y)\cos[\psi(x,y) + 2\pi x\cos\theta/\lambda + \phi_k]$$

much more general than diffuse model.

x, y: image coordinates
I: Intensity
a: ambient light intensity
r: reflectivity, BRDF
ψ: phase shifting
θ: projection angle
ϕ_k: $2\pi(k-1)/3$

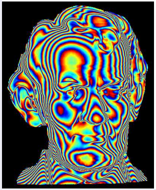


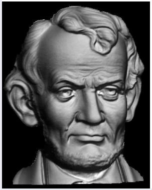
Reconstruction

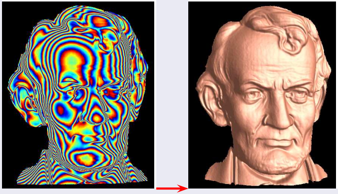
Reconstruction Formula

$$\begin{aligned}\psi &= \tan^{-1}\sqrt{3}\frac{I_0-I_2}{2I_1-I_0-I_2}\\ r &= 2\sqrt{3(I_0-I_2)^2+(2I_1-I_0-I_2)^2}\\ a &= (I_0+I_1+I_2)/3-r/2.\end{aligned}$$

Reconstructed shading $a+r/2$.



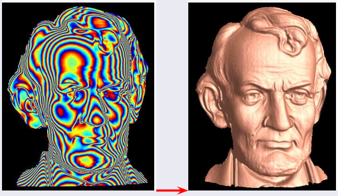




Challenges

Phase Ambiguity The phase reconstructed is from $[-\pi, \pi)$, the reconstructed phase differs from the real phase by $2m\pi$, m is an integer.

Reconstruct Depth from Phase



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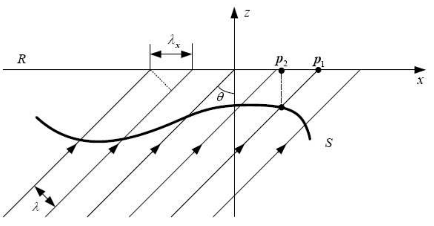
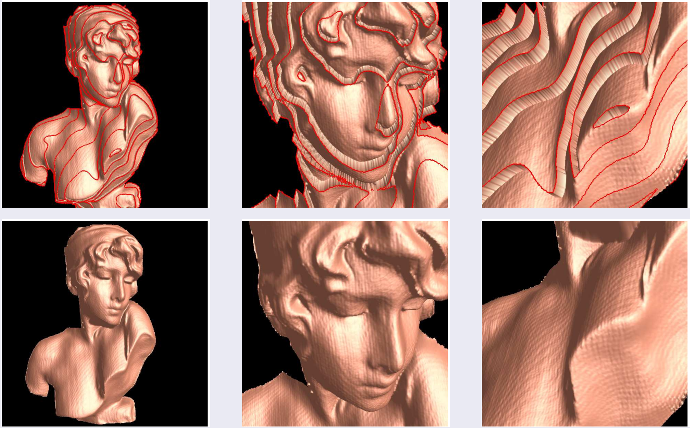
Reconstruct Depth



x: image coordinates
z: depth along optical axis
S : Surface $z(x, y)$
λ: fringe period
θ: projection angle

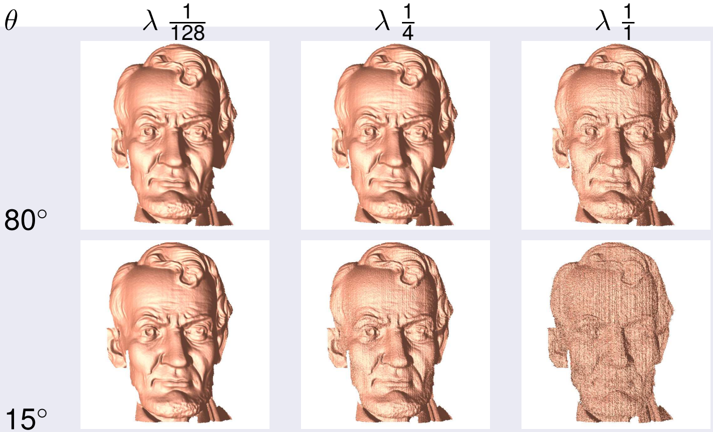
$$z(x, y) = \frac{\psi(x, y)\lambda}{2\pi \sin\theta}$$

Phase Unwrap

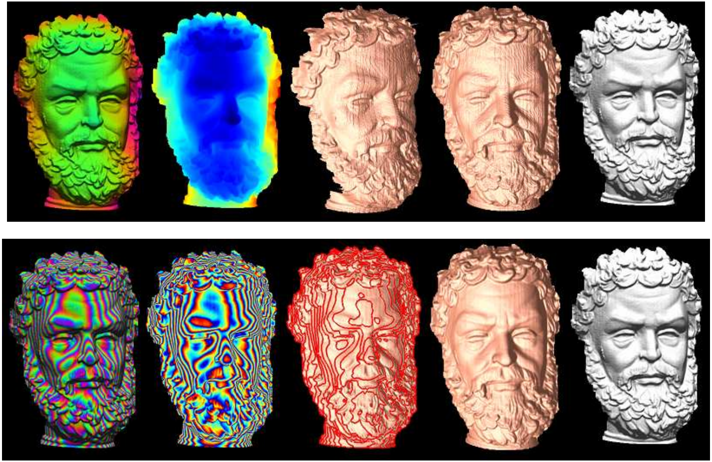


Wrapped geometry. Unwrapped geometry.

Reconstruction Error Analysis



Two wave length phase unwrapping Demo



Reconstructed SurfaceMesh Movie

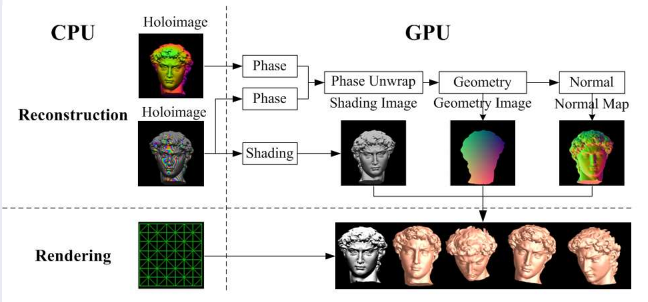
Holoimage Synthesis

Holoimage

Geometry

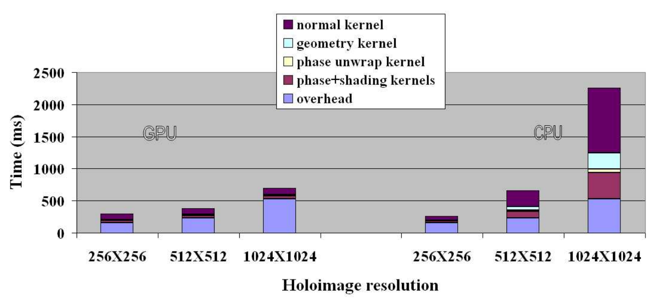
Holoimages on GPU

Pipleline on GPU



Holoimages on GPU

GPU CPU Comparison



Holoimages on GPU

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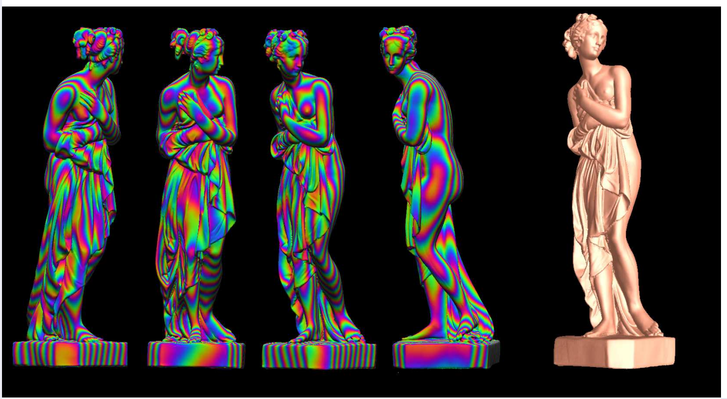
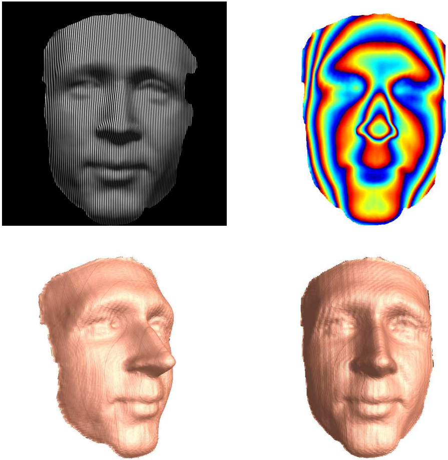


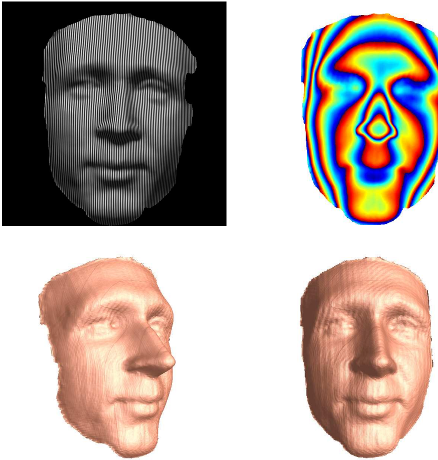
Grayscale Holoimage



Entropy

- Holoimage and geometric surface have the same entropy.
- Conventional image entropy is inappropriate.
- Need to develop new concept on entropy.

Grayscale Holoimage



Entropy

- Holoimage and geometric surface have the same entropy.
- Conventional image entropy is inappropriate.
- Need to develop new concept on entropy.

Grayscale Holoimage

The image intensity is modeled as

$$I(x,y) = a(x,y) + r(x,y)\cos(\psi(x,y) + 2\pi fx)$$

namely

$$I(x,y) = a(x,y) + c(x,y)e^{i2\pi fx} + c^*(x,y)e^{-i2\pi fx},$$

where $c(x,y) = r(x,y)e^{i\psi(x,y)}/2$,$c^*$ is the complex conjugate of c, f is the spacial frequency of the carrier wave.

A one-dimensional Fourier transformation of $I(x,y)$ produces

$$\tilde{I}(\zeta,y) = \tilde{a}(\zeta,y) + \tilde{c}(\zeta - f,y) + \tilde{c}^*(\zeta + f,y),$$

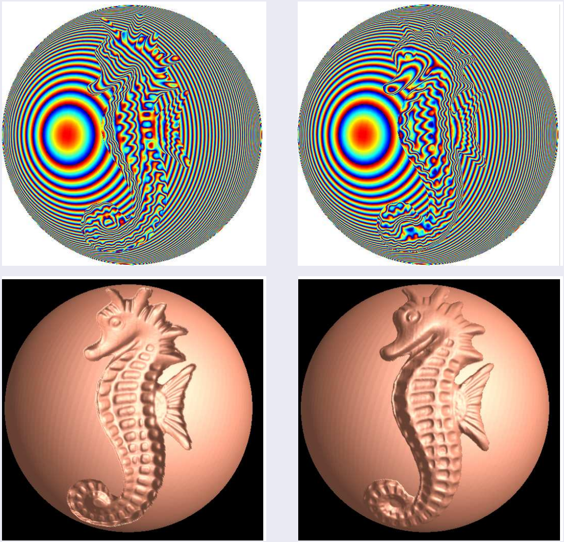
where˜indicate the Fourier transform. If f $\tilde{c}(\zeta - f,y)$ can be obtained by bandpass filter.

Operations between surfaces

By manipulating holoimages, we can compute the sum and difference between their phase maps, then we can accomplish the following geometric operations,

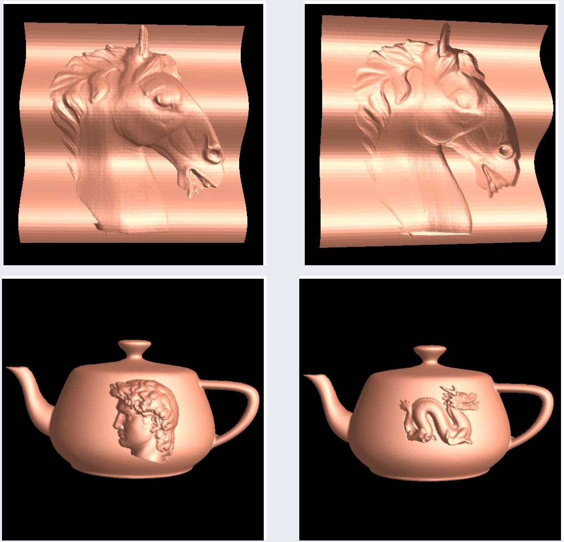
- Embossing
- Engraving
- Extract Geometric Texture
- Measure geometric deformation

Embossing and Engraving

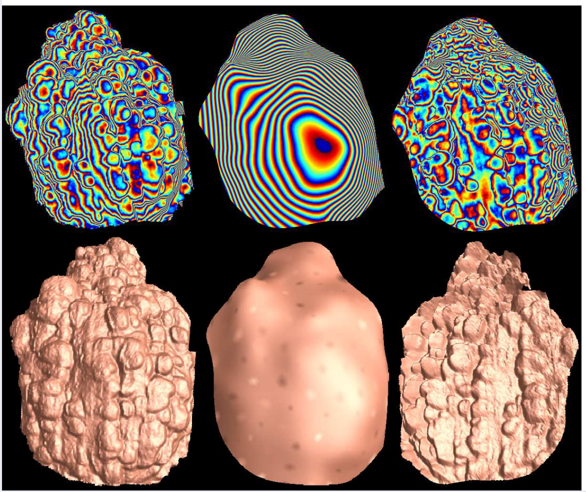


Embossing and Engraving

Embossing and Engraving

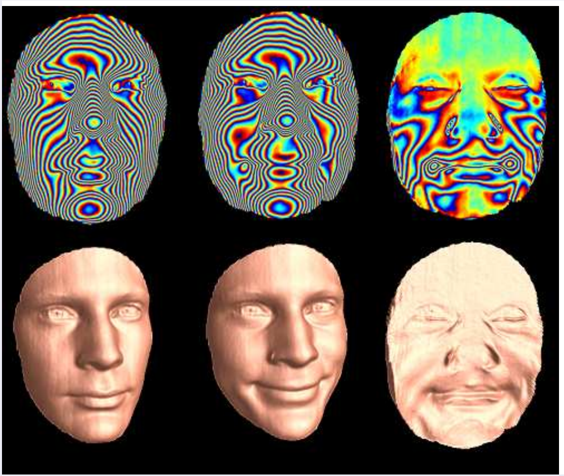


Geometric Texture Extraction



original surface, smoothed surface, geometric texture.

Expression Extraction



calm face, smiling face, the smile without a face

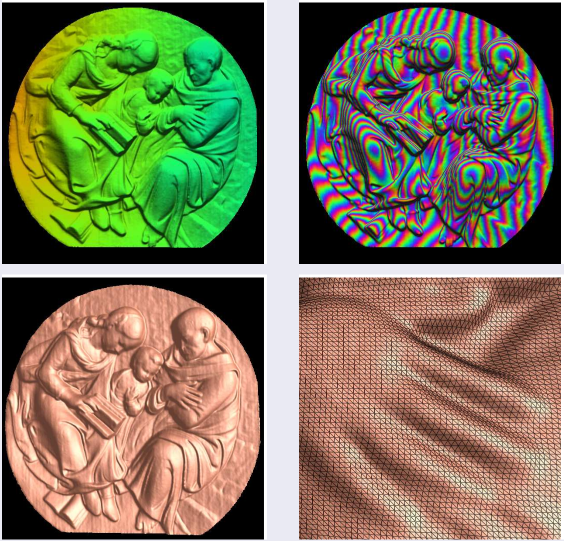
Meshing and Remeshing

Geometric Surface Meshing and Remeshing

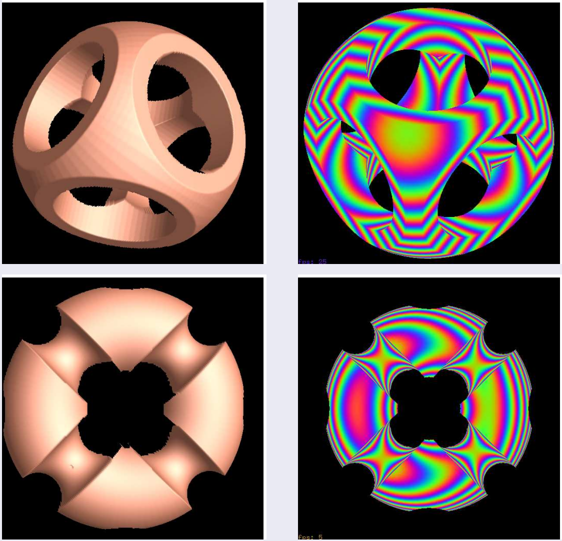
All geometric representations, if they can be rendered, they can be meshed and remeshed using holoimage.

- Point cloud
- Triangle soup
- Implicit surface
- CSG model

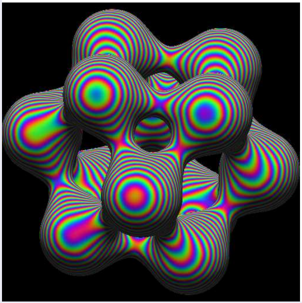
Point Cloud Meshing

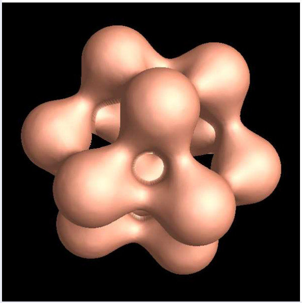


CSG Model Meshing

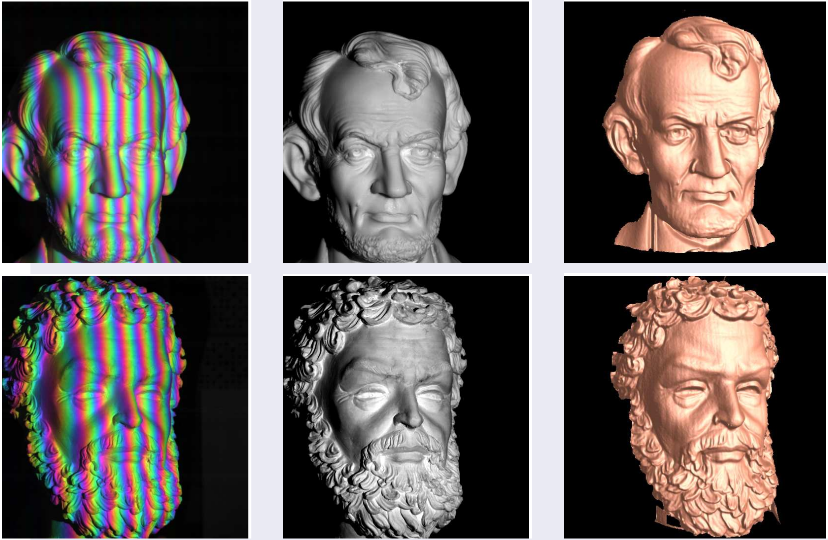


Implicit Model Meshing





Static Geometric Data Acquisition



Static Geometric Data Acquisition

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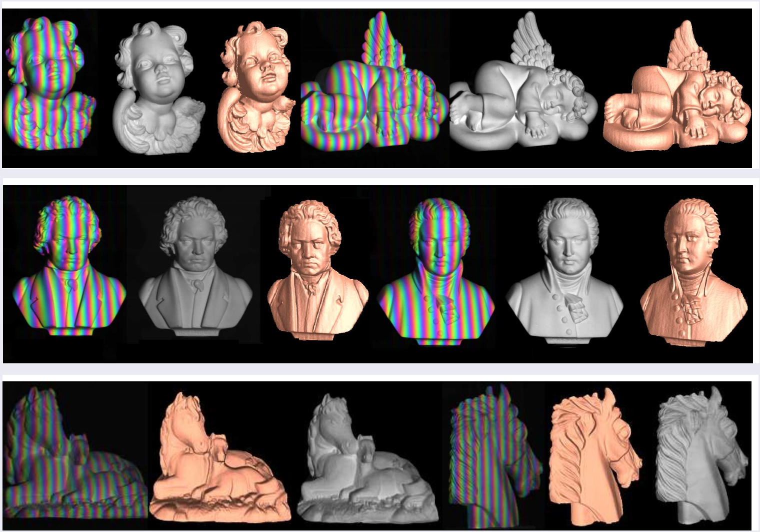
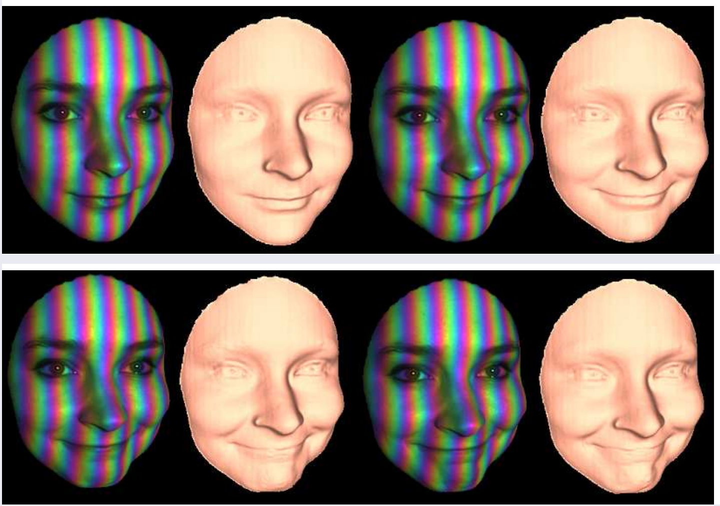
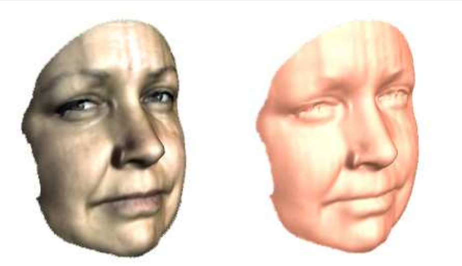


Dynamic Geometric Data Acquisition



Holoimage Geometry

Dynamic Geometric Surface Acquisition with Color Texture

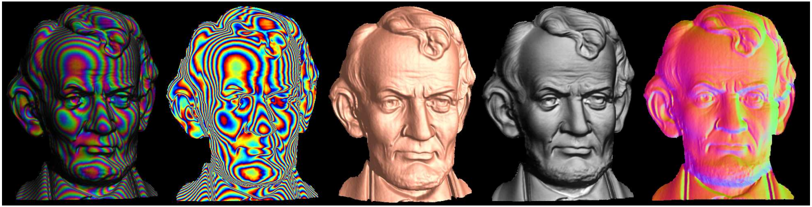


Susan

Summary

1. A novel geometric representation, holoimage. A holoimage encodes both amplitude and phase information, therefore records shading and geometry.
2. Holoimage can be captured from real life by simple setups for high speed geometric data acquisition.
3. Holoimage can be rendered using graphics hardware efficiently.
4. Holoimage can be utilized for many geometric processing tasks.

For more information, please email to gu@cs.sunysb.edu.



Thank you!