CSE 373 Spring 2009

Homework 4 Solutions

6.1

S is the array of numbers and let C_i is the maximum sum of contiguous subsequence ending at S [i]

C_i = $\text{Max}_{j:\, S[j] < S[i]}$ { C_j + S[i] } , if S[j] < S[i] for some j < i

= S[i] , else

Answer is the maximum of all the C_i s

6.6

There are 9 base cases, one for each entry of the table.

C(aa, b) = true; C (ba, c) = true, etc.

We define P [i, j, x] = whether it is possible to parenthesize the substring from index i to j of the given string.

P[i, j, x] = P [i, k, y] && P [k+1, j, z] && C(yz, x) , for any i<=k<j

6.7

If we look at the sub-string x[i,..,j] of the string x, then we can find a palindrome of length at least 2 if x[i] = x[j]. If they are not same then we seek the maximum length palindrome in subsequences x[i+1,...,j] and x[i,...,j-1]. Also every character x[i] is a palindrome in itself of length 1. Therefore base cases are given by x[i,i] = 1. Let us define the maximum length palindrome for the substing x[i,...,j] as $L_{i,j}$

$L_{i,j}$ = $L_{i+1,j-1} + 2$,if x[i] = x[j]
= $\max\{L_{i+1,j}, L_{i,j-1}\}$,otherwise

$L_{i,i}$ = 1 ,For all $i \in (1, ..., n)$

6.8

L [$X_{1..i}$, $Y_{1..j}$] = L [$X_{1..i-1}$, $Y_{1..j-1}$] + 1 , if X[i] = Y[j]
= 0 , else
Find the maximum entry in the m x n matrix

6.11

Define $d_{i,j}$ to be the length of the longest common subsequence of X[1..i] and Y [1..j]. Let D be the n × m matrix [$d_{i,j}$].

$d_{i,j}$ = 0 , if i=0 or j=0
= $d_{i-1,j-1} + 1$, if X[i]=Y[j]
= Max { $d_{i-1,j}$, $d_{i,j-1}$ } , else

Answer is $d_{m,n}$

6.17

Let C[v] be the Boolean value that says whether it is possible to change for v cents using coins of denominations D = {d_1, d_2, . . . , d_n }.

C[v] = true ,if $v \in D$
= C[v – d_i] for any $j \in \{1..n\}$,else

6.18

Let C[v, D] be the Boolean value that says whether it is possible to change for v cents using coins of denominations D = {d_1, d_2, . . . , d_n } (one coin at most once).

C[v, D] = true ,if $v \in D$
= C[v – d_i, D- {d_i}] for any $j \in \{1..n\}$,else

6.19

Let C[v, k] be the Boolean value that says whether it is possible to change for v cents using only k coins from denominations D = {d_1, d_2, . . . , d_n }.

C[v, k] = true ,if $v \in D$
= C[v – d_i, k-1] for any $j \in \{1..n\}$,else

6.22

Let us define S(i, t) = some subset of X[i .. n] sums to t.

Our goal is to compute S(1, T), using the recurrence

S(i, t) = TRUE if t = 0,
= FALSE if t < 0 or i > n,
= S(i + 1, t) ∨ S(i + 1, t − X[i]) otherwise

If S(i + 1, t) and S(i + 1, t − X[i]) are already known, we can compute S(i, t) in constant time, so memorizing this recurrence gives us and algorithm that runs in $O(nT)$ time.

6.25

Let us define $f(i, t_1, t_2, t_3)$, which is true iff there is a partition of $\{a1, a2 ... a_i\}$ into three sets that add up to t_1, t_2, t_3 respectively. The i-th element may belong to the first, the second or the third partition.

$f(i, t_1, t_2, t_3) = f(i-1, t_1-a_i, t2, t3) \vee f(i-1, t_1, t_2-a_i, t_3) \vee f(i-1, t_1, t_2, t_3-a_i)$.

If the sum of all a_i is S, then the answer is $f(i, S/3, S/3, S/3)$.