

CSE 373 Midterm (100 points)

2nd April: 3:50-5:10pm

Open 1-Sheet.

Instructions: Write only precise answers (with minimal scratch work) in the space provided for answers. For additional scratch work, you may use the back of the pages. **Please write your name below.**

Name:

1. **Asymptotic Notation (20 points).**

Prove that $(n^3 - 3n^2 - n + 1) = \Theta(n^3)$.

2. Searching (25 points).

Suppose that you are given a *sorted* sequence of *distinct* integers $\{a_1, a_2, \dots, a_n\}$, drawn from 1 to m where $n < m$. Give an $O(\log n)$ algorithm to find an integer $\leq m$ that is not present in the given sequence. For full credit, find the smallest such integer.

3. **Graphs (25 points)**. A mother vertex in a directed graph $G = (V, E)$ is a vertex v such that all other vertices in G can be reached by a directed path from v .
- (a) Give an $O(|V| + |E|)$ algorithm to test whether a given vertex v is a mother of G .
 - (b) Give an $O(|V| + |E|)$ algorithm to test whether a graph G contains a mother vertex, if G has *no directed cycles*.

4. **Distance in Trees (30 points).**

The diameter of a tree $T = (V, E)$ is given by

$$\max_{u, v \in V} \delta(u, v)$$

where $\delta(u, v)$ is the number of edges on the path from u to v . Describe an efficient algorithm to compute the diameter of a tree, and show the correctness of the algorithm, and the time complexity. **Hint:** Go from leaves to the root, keeping track of something (what?) at each node. There is a linear-time algorithm for this. For almost-full credit, you may assume the tree to be binary.