CSE 373 Midterm (100 points)
1st November: 5:20pm-6:40pm
Open 1-Sheet.

Instructions: Write only precise answers (with minimal scratch work) in the space provided for answers. For additional scratch work, you may use the back of the pages. **Please write your name below.**

Name:

1. **Asymptotic Notation (20 points)**.

   Prove that $(n^3 - 1000n^2) = \Omega(n^3)$.

2. **Sorting (20 points)**.
Suppose that you are given a *sorted* sequence of *distinct* numbers $\{a_1, a_2, \ldots, a_n\}$. Give an $O(\lg n)$ algorithm to determine whether there exists an index i such that $a_i = i$. For example, in $\{-10, -3, 3, 5, 7\}$, $a_3 = 3$. In {2,3,4,5,6,7}, there is no such i. A pseudo-code with brief explanation will be perfect.

3. **Greedy Algorithm (20 points)**.

Intervals with Profits. You are given a set of n jobs/intervals $\{(s_i, f_i, q_i)\}$, where s_i is the start time of the job, f_i is the finish time, and q_i is the profit. Our goal is to find the maximum total profit possible using a set of non-overlapping jobs.

Consider the following greedy algorithm. First, sort the jobs so that $q_1 \geq q_2 \geq \ldots q_n$. Then, try to add each job to the schedule in turn. If job i does not conflict with any jobs scheduled so far, add it to the schedule; otherwise, discard it. Add up the profits of the jobs scheduled in the above manner.

Give a counterexample to show that the above given algorithm may not return an optimal profit.

4. **Dynamic Programming (40 points).** Let us design a dynamic programming algorithm for the above problem. First, sort the jobs so that $f_1 \leq f_2 \leq \ldots f_n$. Now, let S_i be the maximum total profit possible using a set of non-overlapping jobs from the first i jobs (i.e., from the i jobs with finish time less than or equal to f_i).

   Based on the above idea, answer/write the following.

   (a) Write a recursive equation to compute S_i in terms of S_j where $j < i$.

   (b) Using the above recursive equation, give the overall psuedo-code of the suggested dynamic programming algorithm.

   (c) What is the time complexity of the dynamic programming algorithm? Briefly explain.