Joint Routing, Channel Assignment, and Scheduling for Throughput Maximization in General Interference Models

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Abstract—Throughput optimization in wireless networks with multiple channels and multiple radio interfaces per node is a challenging problem. For general traffic models (given a set of source-destination pairs), optimization of throughput entails design of "efficient" routes between the given source-destination pairs, in conjunction with (i) assignment of channels to interfaces and communication links, and (ii) scheduling of non-interfering links for simultaneous transmission. Prior work has looked at restricted versions of the above problem. In this article, we design approximation algorithms for the joint routing, channel assignment, and link scheduling problem in wireless networks with general interference models. The unique contributions of our work include addressing the above joint problem in the context of physical interference model and single-path routing (wherein, traffic between a source-destination is restricted to a single path). To the best of our knowledge, ours is the first work to address the throughput maximization problem in such general contexts. For each setting, we design approximation algorithms with provable performance guarantees. We demonstrate the effectiveness of our algorithms in general contexts through simulations.

Index Terms—C.2.2 Network Protocols, F.2 Analysis of Algorithms and Problem Complexity.

1 Introduction

One of the central questions in communication networks is: Given a set of source-destination pairs, what is the maximum rate (throughput) at which the network can transfer data from the sources to the corresponding destinations? The above throughput maximization problem is challenging in wireless network due to the presence of wireless interference. In addition, one can significantly increase the network throughput by equipping each node with multiple radio interfaces that operate on orthogonal channels. Availability of multiple channels and interfaces poses the additional challenge of determining efficient assignment of channels to links and interfaces. In general, the throughput maximization problem in wireless networks entails jointly optimizing routing, channel assignment (to interfaces and links), and interference-free scheduling of links.

Prior works on the above throughput maximization problem have only addressed restricted versions of the problem: single channel [9, 22], static channel assignment [2], channel assignment and scheduling for a predetermined set of possible paths [23]. Finally, [19] derives upper bounds on the achievable throughput, without designing approximation algorithms. The main shortcomings of the prior works are three fold. *Firstly*, [2] considers only a static assignment of channels to interfaces. On the other hand, dynamic channel assignment offers more flexibility and improved capacity [19], and incurs minimal overhead using improved hardware technology [23]. The work in [23] does consider dynamic channel assignment, but for a predetermined set of paths between each source-destination pair. *Secondly*, all the prior works[1] on throughput maximization are for simple (pairwise) interference models, wherein the model is represented as a set of *pairs* of links that interfere with each other. On the other hand, the physical interference model is less restrictive, and in general yields higher capacity than pairwise interference model in scenarios that do not use CSMA techniques [5]. *Thirdly*, all the prior works on throughput maximization consider multi-path routing (or predetermined single-path routing [23]) between each source-destination pair. Multipath internetworks are more complex to configure, while single-path routing infrastructure has simplified routing tables. Moreover, in single-path routing, the problem of packet-reordering (needed in multi-path routing) does not exist. Indeed, in conventional networks (e.g., Internet), application-level flows generally use single-path routing.

Motivated by the above considerations, in this article, we address the joint routing, channel assignment, and scheduling problem for throughput maximization (hereafter, refereed to as the *JRCAS problem*) in wireless networks in the following general contexts: (i) multiple channels with dynamic channel assignment, (ii) physical interference model,

1. In a recent concurrent work, [9] considers throughput maximization in physical-interference for single-channel networks. The approximation results in [9] are based on a certain assumption, to incorporate presence of noise (see Section 2 for details).

and (iii) single-path routing. In particular, our main contributions are:

- For pairwise interference (Section 3), we present a $(c+2)$-approximation algorithm for the JRCAS problem with *dynamic* channel assignment. Here, c is the network-interference degree [10, 23], a small constant which depends on the interference model.
- For physical interference model (Section 4), we design two constant-factor approximation algorithms for the JRCAS problem. Our first algorithm is an improvement and generalization of the result in [9]. Our more significant contribution, the second approximation algorithm, is based on a novel linear-equation representation of the physical interference constraint, and has a constant-factor approximation bound independent of the transmission power and noise values.
- The most significant contribution of our work is for the JRCAS problem with single-path routing (Section 5) for which we design randomized approximation algorithms for both pairwise and physical interference models, using the classic rounding technique [29].

Our techniques also yield the following result.

- For the TDMA link scheduling problem [14] in multi-channel multi-radio networks with physical interference, we design two constant-factor approximation algorithms (Section 4.3).
- For the JRCAS problem with static channel assignment, we get constant-factor approximation algorithms for both pairwise and physical interference models. For the pairwise interference model, our algorithm is *much* simpler than the very involved result of [2] from MobiCom 2005.

2 Models, Problem Description, and Related Work

Network Model. A wireless network is modeled as a directed graph $G(V,E)$, where V is the set of network *nodes* and E is the set of *directed* communication *links* each connecting a pair of nodes. A directed link (u,v) denotes that u can transmit to v directly (in absence of other interfering transmissions). Link capacity $\kappa(e)$ of a link e is the maximum data rate (bits/sec) that can be carried on e. There are K orthogonal channels available, and each node u is equipped with $I(u)$ (radio) *interfaces*. We use the notation $N(u)$ to denote the set of links incident on node u. That is,

$$N(u) = \{e | e = (u,v) \text{ or } (v,u), \text{ and } e \in E\}. \quad (1)$$

<u>Interference Models.</u> Due to the broadcast nature of the wireless links, transmission along a link may interfere with other link transmissions, when transmitted on the same channel (links on different channels do not interfere). An *interference model* defines which set of links can be active simultaneously without interfering. We consider two types of interference models, viz., pairwise and physical. A pairwise interference model is represented by a set of pairs of links that interfere with each other. The pairwise model can be represented by a conflict graph, wherein the vertices are the links and the edges identify pairs of interfering links. In the physical interference model, successful transmission over a link (u,v) depends on the signal-to-noise ratio (SINR) at v.

<u>Time Slots.</u> In our model, the system operates synchronously in a time slotted model. In any time slot, a set of non-interfering links are active, and each interface is assigned a channel. In the *static channel assignment* model, the assignment of channels to interfaces is fixed across time slots, while in the *dynamic assignment* model, an interface can choose different channels in different time slots. Each active link (u,v) uses a pair of interfaces (assigned the same channel) at u and v. We assume unicast transmissions; thus, an interface can be used by at most one link. However, in a time slot, a link (u,v) may support multiple simultaneous transmissions using multiple pairs of interfaces. Thus, in a time slot, a *multiset* of links may be active, with each *instance* of a link associated with a different pair of interfaces.

JRCAS Problem. The JRCAS (joint routing, channel assignment, and scheduling) problem to maximize throughput can be informally described as follows; see next section for a more formal description. <u>Input</u>: A wireless network graph, the interference model, and a set of source-destination pairs. <u>Output</u>: An interference-free schedule of link transmissions into time slots that guarantees maximum total data rate between the given source-destination pairs. By default, we assume multi-path routing. In either case, the flows must observe link-capacity, flow conservation, and "interface" constraints. <u>Notes.</u> (i) Design of a link schedule entails assignment of channels to link instances. (ii) The JRCAS problem is a generalization of the classic multicommodity flow problem [1] with additional resources (channels and interfaces), constraints (interference and interface), and outputs (link schedule and channel assignment).

Related Work. One of the first works that addressed the throughput maximization problems is the work by Jain et al. [16], where the authors give an LP formulation of the problem. However, their formulation requires enumeration of all interference-free sets of links, which can be exponential in number. The above shortcoming was first remedied in an insightful work by Kumar et al. [22], who design an approximation algorithm for interference-free scheduling of links for throughput maximization in single channel networks with transmitter interference model. Our work builds on their key insight. Alicherry et al. [2] address the JRCAS problem with IEEE 802.11 MAC-based interference and static channel assignment, and design a $(cK/I_{\min})$-approximation algorithm. Here, c is the network-interference degree, K is the number of channels, and $I_{\min}(\leq K)$ is the minimum number of interfaces per node. However, their approach is unnecessarily involved and rather complicated. In fact, in this article, we derive the same approximation bound with a trivial generalization of [22]'s work. In a recent work, [7] present an improvement of the above works ([2, 22]). However, their work has a fundamental flaw [32]; in particular, their claim (without proof) that the resulting flows can be scheduled is *incorrect*. See Figure 1 for a counter example.

Finally, the recent concurrent work [9] by Chafekar et al. considers the JRCAS multi-path problem for physical interference model in single channel networks. In addition to the restrictions of single channel and multi-path networks, the approximation results in [9] are based on the assumption that the optimal algorithm is restricted to use a slightly smaller transmission power (and hence only a subset of the links of the original communication graph are usable by the optimal algorithm). The assumption is used to avoid making the assumption of zero noise (as in [14]); see Section 4.1 for details. In contrast, in this article, we do not make any such assumptions.

Another line of related research stems from the seminal work by Tassiulas and Ephremides [33], who present an optimal link scheduling policy for arbitrary network models. However, their scheduling policy needs to iteratively solve an optimization problem (maximum-weighted interference-free set of links) that is NP-hard even for simple interference models. Also, their scheme has not been extended to solve the *joint* routing and scheduling problem. Based on the above result, [10, 25, 31] design simple scheduling policies that guarantee near-optimal throughput for single channel. Recently, [23, 30] extend the above ideas and consider the joint (dynamic) channel assignment and scheduling problem, for a predetermined set of possible paths. Another recent work [13] extends the ideas of [33] to include fairness. Taking a different approach to extend [33]'s work, the authors of [6] use a notion of *local pooling* to design efficient channel assignment algorithms under primary interference model. All the above extensions are for pairwise interference.

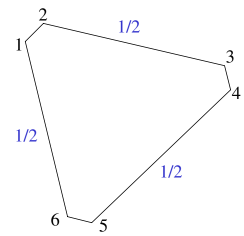

Fig. 1. Counter-example for [7]'s claim. In the shown network graph, all the links have unit capacity. Assume secondary (802.11 based) interference model. Let the link utilizations (the fraction of times they are active) of the directed links (1,6), (5,4), and (3,2) be 1/2 units, and on all other links be 0. For the given instance, the "sufficiency" condition[3] of [7] is satisfied, but *no schedule is possible* since the above three links are mutually conflicting and no schedule can have each one of them operating 1/2 of the time.

Chen et al. [11] consider jointly optimizing congestion control, routing, and scheduling for networks with single channel and pairwise interference, and design an approximation algorithm. Their work is based upon (and extension of) earlier works of Lin et al. [24] and Neely et al. [28].

Other Works. The TDMA link scheduling problem has been addressed before [5, 14, 34], but is a special case (in terms of designing approximation schemes) of the JRCAS problem as shown in Section 4. Recently, Chafekar et al. [8] addressed the problem of minimizing end-to-end delay for *one packet per source-destination* [21] by jointly optimizing routing, power control, and scheduling for physical interference. The objective of our JRCAS problem is different than theirs. In other works, [27] addresses the joint scheduling and power control problem, [4] considers the joint routing and scheduling problem for power optimization, and [18] considers a class of scheduling problems without addressing the interference constraint.

3. The sufficiency condition of [7] essentially states that for every node u, the sum of fractions of time the neighboring nodes of u (including u) are active must be bounded by 1. More formally, if $\mathcal{N}(u)$ denotes the set of 1-hop neighbors of u, and τ_u and τ_{uv} denote the fractions of time node u and link (u, v) are active respectively, then $\sum_{v \in \mathcal{N}(u) \cup \{u\}} \tau_v - \sum_{v,w \in \mathcal{N}(u) \cup \{u\},\, (v,w) \in E} \tau_{uv} \leq 1$. While the above is a necessary condition, it is clearly not sufficient as the example shows.

3 JRCAS Problem with Pairwise Interference

We start with a few definitions here.

Definition 1: (Pairwise Interference Model; Conflict Graph; $C(e)$) The *pairwise interference model* is represented by a set of *pairs* of links that interfere with each other, if active on the same channel.

The set of pairs of interfering links is represented by a *conflict graph* $G(V_c, E_c)$. The set of vertices V_c of a conflict graph are the network links, and the set of edges E_c connect the pairs of vertices that correspond to interfering links. We use $C(e)$ to denote:

$$C(e) = \{e\} \cup \; \{e' | (e, e') \in E_c\} \tag{2}$$

Most interference models, e.g., transmitter model [35], protocol model [15], transmitter-receiver model [3], etc., can be modeled as pairwise interference. □

Definition 2: (Network-Interference Degree.) *Network-interference degree* is the maximum interference degree of any link, where the interference degree of a link (u, v) is the number of links that interfere with (u, v) but not with each other. In other words, *network interference degree* is the size of the maximum independent set in the subgraph induced by the neighbors of any vertex in the conflict graph.

Network-interference degree is generally dependent only on the interference model (independent of the network topology). For example, the simple node-exclusive (primary) interference model has a network-interference degree of 2, while the uniform-range secondary interference model which approximates IEEE 802.11 DCF has a network-interference degree of 8 [10]. □

Definition 3: (Link Schedule.) A *link schedule* is a specification of a certain number of time slots. For each time slot, we specify a *multiset* of active links with a channel assigned to each link instance. A valid link schedule must satisfy two constraints: (a) The link instances active in the same time slot do not interfere, and (b) the number of different channels incident on any node u in a time slot is less than $I(u)$. □

Definition 4: (Link Utilizations, $\alpha(e, k)$ and $\alpha(e)$.) *Link utilization* $\alpha(e, k)$ of a link e for channel k in a given link schedule is the ratio of the (i) total number of instances (across all time slots) of link e active on channel k, and (ii) the total number of slots of the link schedule. Note that the first term is cumulative across all source-destination pairs. Also, we use $\alpha(e)$ to denote $\sum_k \alpha(e, k)$. □

Definition 5: (Link Flows, $f(e, k), f(e), f_i(e, k), f_i(e)$.) *Link flow* $f(e, k)$ for a link e and channel k is the data rate carried by link e on channel k, i.e., $f(e, k) = \kappa(e)\alpha(e, k)$. We use $f(e)$ to denote $\sum_k f(e, k)$, and $f_i(e, k)$ or $f_i(e)$ to denote the portion of the link flow for a particular source-destination pair $\{s_i, d_i\}$. Thus, $f(e, k) = \sum_i f_i(e, k)$ and $f(e) = \sum_i f_i(e)$. □

Based on the above definitions, we now give a formal description of our JRCAS problem.

JRCAS Problem with Pairwise Interference. Given a network graph, its conflict graph, and a set of source-destination pairs, the JRCAS problem is to design a link schedule that maximizes the total data rate between the given source-destination pairs. The resulting link flows must satisfy the flow conservation constraints (formally given by Equations 6-8 later) at each node. Note that a link schedule by definition includes assignment of channels to active link instances, and satisfies interference and interface constraints. The above JRCAS problem is NP-hard, since the special case TDMA link scheduling problem [34] is NP-hard.

Overview of General Approach. Using [22]'s approach, we start with a linear programming (LP) formulation of the JRCAS problem that incorporates interface and interference constraints, and constraints relating link flows, link capacities, and link utilizations. LP is solved optimally in polynomial time. However, the LP solution only gives the link flows, and not a link schedule that realizes the obtained link flows. But, we can design a near-optimal link schedule as follows. First, we scale down the link utilizations obtained from the LP solution by a certain factor, to satisfy a certain "sufficiency" condition which allows us to design a link schedule for the scaled-down link utilizations. Since, the total data rate of the LP solution is an upper bound on the optimal, the above yields an approximate link schedule.

We start with describing the single-channel solution, which is a slight generalization (without predetermined routes) of [22]'s result. Then, we generalize the technique to multiple channels.

JRCAS Problem for Single Channel. We start with our LP formulation. We use i to vary over given source-destination pairs $\{s_i, d_i\}$, F_i to denote the data rate between s_i and d_i, and V_i to denote $V -$

$\{s_i, d_i\}$. $N(u)$ is as defined before (Equation 1).

$$\forall\ i, \qquad F_i \geq 0 \tag{3}$$
$$\forall\ i \neq j, \qquad F_i \geq \psi F_j \tag{4}$$
$$\forall\ i, e \in E, \qquad f_i(e) \geq 0 \tag{5}$$
$$\forall\ i, u \in V_i, \sum_{(v,u)\in E} f_i((v,u)) = \sum_{(u,w)\in E} f_i((u,w)) \tag{6}$$
$$\forall\ i, \quad \sum_{(w,s_i)\in E} f_i((w,s_i)) + F_i = \sum_{(s_i,v)\in E} f_i((s_i,v)) \tag{7}$$
$$\forall\ i, \quad \sum_{(v,d_i)\in E} f_i((v,d_i)) = \sum_{(d_i,w)\in E} f_i((d_i,w)) + F_i \tag{8}$$
$$\forall\ e \in E, \qquad \alpha(e) = \sum_i f_i(e)/\kappa(e) \tag{9}$$
$$\forall\ u \in V, \qquad \sum_{e\in N(u)} \alpha(e) \leq 1 \tag{10}$$
$$\text{Maximize} \qquad \sum_i F_i \tag{11}$$

Above, Equation 4 represents the fairness constraint [22], ensuring that the ratio between the minimum and maximum data rates does not go below a given constant ψ. Equations 6-8 represent flow conservation constraints, and Equation 10 represents the interface constraint.

Interference Constraint. We need to incorporate interference constraint in the above LP. Consider a time slot t, and let X_e represent the binary variable which is 1 if the e is scheduled in t and is 0 otherwise. Then, for any link e, $\sum_{e'\in C(e)} X_{e'} \leq c$, where c is the network-interference degree and $C(e)$ is as defined before (Equation 2). Averaging the above over all time slots, we get:

$$\sum_{e'\in C(e)} \alpha(e') \leq c, \quad \forall\ e \in E. \tag{12}$$

We *add the above equation* to the LP.

Near-Optimal Link Schedule. As mentioned before, the LP solution does not give a link schedule. In fact, there may not exist any link schedule that guarantees the link utilizations of the LP solution (see Figure 1).[4] Let the link utilizations obtained from the LP solution be $\{\widehat{\alpha}(e)\}$, and let $\overline{\alpha}(e) = \widehat{\alpha}(e)/c$ for each e. Now, for each e, $\sum_{e'\in C(e)} \overline{\alpha}(e') \leq 1$. Based on this inequality, a link schedule S that realizes the $\overline{\alpha}$ link utilizations can be easily designed [22]. Also, the total data rate due to S is at least $1/c$ of the optimal possible, since the optimal data rate is at most that of the LP solution. We prove the above in a more general context (see Theorem 2).

4. Even if we add an interference constraint in the LP for each *clique* in the conflict graph, the link utilization returned by LP solution may still not be realizable by a link schedule ("extend" Figure 1 to ten network nodes [16]).

Multiple Channels with Static Channel Assignment. The above single-channel solution also yields a $(cK/I_{\min})$-approximate solution for multiple channels with static channel assignment. Here, K is the number of channels and $I_{\min}(\leq K)$ is the minimum number of interface per node. First, note that any single-channel link schedule S can be transformed into a multi-channel link schedule S' with a total data rate of $I_{\min}$ times that of S, by using $I_{\min}$ interfaces per node.[5] Second, note that the optimal data rate with K channels is at most K times the optimal data rate with one channel. Based on the above two observations, the c-approximate single-channel solution can be transformed into a $(cK/I_{\min})$-approximate solution for multiple channels with static channel assignment. The above is a much simpler result than that of [2].

Theorem 1: The above algorithm gives a $(cK/I_{\min})$-approximate solution to the JRCAS problem with multiple channels and static assignment of channels. Here, c is the network-interference degree, K is the number of channels, and $I_{\min}$ is the minimum number of interfaces per node. ∎

Multiple Channels with Dynamic Channel Assignment. The LP formulation for the case of dynamic channel assignment is shown below. We use k to vary over available channels.

$$\forall\ i, \qquad F_i \geq 0 \tag{13}$$
$$\forall\ i \neq j, \qquad F_i \geq \psi F_j \tag{14}$$
$$\forall\ i, k, e \in E, \qquad f_i(e,k) \geq 0 \tag{15}$$
$$\forall\ i, e \in E, \qquad f_i(e) = \sum_k f_i(e,k) \tag{16}$$
$$\text{Flow conservation Equations 6 to 8} \tag{17}$$
$$\forall\ k, e \in E, \qquad \alpha(e,k) = \sum_i f_i(e,k)/\kappa(e) \tag{18}$$
$$\forall\ u \in V, \qquad \sum_{e\in N(u)} \sum_k \alpha(e,k) \leq I(u) \tag{19}$$
$$\forall\ k, e \in E, \qquad \sum_{e'\in C(e)} \alpha(e',k) \leq c \tag{20}$$
$$\text{Maximize} \qquad \sum_i F_i \tag{21}$$

Above, Equations 19 and 20 represent the interface and interference constraint respectively.

Near-Optimal Link Schedule. As before, we first solve the above LP optimally. Let $\{\widehat{\alpha}(e,k)\}$ be

5. Essentially, any $I_{\min}$ time slots of S can be combined into one time slot.

the link utilizations of the LP solution, and let $\overline{\alpha}(e,k) = \widehat{\alpha}(e,k)/(c+2)$ be the new scaled-down link utilizations. For any link $e = (u,v)$, let

$$\eta(\overline{\alpha}, e) = \frac{1}{I(u)} \sum_{e' \in N(u)} \overline{\alpha}(e') + \frac{1}{I(v)} \sum_{e' \in N(v)} \overline{\alpha}(e'), \quad (22)$$

where $\overline{\alpha}(e') = \sum_k \overline{\alpha}(e',k)$. Then, it is easy to see that the $\overline{\alpha}$ values satisfy the following *sufficiency* condition:

$$\forall\ e,k, \quad \eta(\overline{\alpha}, e) + \sum_{e' \in C(e)} \overline{\alpha}(e',k) \leq 1 \qquad (23)$$

Based on the above, we can now design a link schedule for the $\overline{\alpha}$ link utilizations as follows.

- Pick a large enough integer W such that $W\overline{\alpha}(e,k)$ is a positive integer for each e and k.
- Consider a link schedule S of W time slots.
- Iterate through all pairs (e,k) in an arbitrary order, and place the link e with channel k in the first $\overline{\alpha}(e,k)W$ time slots of S, wherein such a placement does not cause any interference with or violate interface constraint due to previously placed link instances. We refer to the above as the *greedy placement* algorithm. The below theorem shows that such a placement of links into S is always possible.

Theorem 2: The above algorithm returns a $(c+2)$-approximate link schedule for the JRCAS problem with dynamic channel assignment.

Proof: In the above described algorithm, for any pair (e,k), the maximum number of time slots wherein (e,k) can *not* be placed due to interference with previously placed links is $W \sum_{e' \in C(e)\setminus\{e\}} \overline{\alpha}(e',k)$ and due to interface constraint violation is $W\eta(\overline{\alpha}, e)$. Thus, by Equation 23, the pair (e,k) *can* be placed in at least $W\overline{\alpha}(e,k)$ time slots of S. Thus, the above described algorithm delivers a link schedule S of W time slots with link utilizations $\overline{\alpha}(e,k)$ for each link e and k. Since $\overline{\alpha}(e,k) = \widehat{\alpha}(e,k)/(c+2)$, the link schedule S delivered by the above algorithm has a total data rate of $1/(c+2)$ times the total data rate of the LP solution. Since the optimal data rate is bounded by the data rate of the LP solution, the designed link schedule S is a $(c+2)$-approximate solution. Note that scaling down of the link utilizations does not violate any LP constraint. ∎

Improved Bounds by Ordering Links. In our above algorithm to place links in a schedule, we considered links in arbitrary order. However, considering links in a certain order (depending on the network model) can sometimes result in improved approximation bounds. For instance, for networks with non-uniform interference range, the network-interference degree may be unbounded. But, considering links in the order of their ranges as suggested in [22] results in a constant-factor approximation scheme. Also, for networks with the IEEE 802.11 based secondary interference model with uniform transmission range, the best-known approximation bound for *single-channel* JRCAS problem is 8 [2, 22], since the network-interference degree c is 8 [10]. This can be improved to 6 if special link ordering is used [17].

Generalizations. Techniques of this section easily generalize to (i) directional antenna, (ii) multiple transmission powers, and (iii) more constraints and objective functions. Directional antenna can be handled by defining "flavors" of each link (u,v) — each flavor corresponds to a "feasible" pair of directions of antennas at u and v. A conflict graph is then constructed over $(link, flavor)$ as vertices, and our techniques can then be applied. Multiple transmission powers can be handled similarly; however, as suggested in the previous paragraph, we may need to consider links in a certain order. Finally, we can add any constraint to the JRCAS problem that is preserved by scaling down of the link utilization by a constant factor. For instance, for *given* traffic demands T_i for each source-destination pair, we can consider the objective of minimizing the scaling factor λ such that T_i/λ data rates can be satisfied. For above, we can just replace Equation 14 by $F_i = T_i\lambda$ and use the same techniques. Similarly, our techniques (and approximation proofs) will work for any objective function that is a linear combination of link flows.

4 JRCAS Problem with Physical Interference

In this section, we address the JRCAS problem with physical interference. As mentioned before, physical interference is less restrictive, and in general entails more capacity than pairwise model in scenarios that do not use CSMA techniques [5]. In the physical interference model, if $P_v(x)$ denotes the received power at v of the signal transmitted by node x, then a packet along link (u,v) is correctly received if and only if:

$$\frac{P_v(u)}{N + \sum_{w \in V_*} P_v(w)} \geq \beta,$$

where N is the background noise, V_* is the set of nodes that are transmitting simultaneously, and

β is a SINR constant.[6] Below, we present two approximation algorithms for the JRCAS problem with physical interference.

4.1 Approximation Algorithm Based on Length Classes

Our first approximation algorithm for the JRCAS problem with physical interference, is based on techniques from [14].[7]

We start with making the following assumptions.

- For now, we assume that zero background noise ($N = 0$); we will remove this assumption later.
- We assume that the radio signal propagation obeys the log-distance path model with path loss exponent γ, commonly assumed in the literature to be greater than 2 [15]. In other words, the signal strength at a distance d from a node transmitting with a power of P is assumed to be equal to P/d^γ.
- We assume fixed and uniform transmission power P at each node. Transmission power control can be achieved on top of our techniques using techniques similar to [9]; we omit the details in this article.

Length Classes and Grid Cells. Using the notations of [14], let *length class* L_j denote the set of links whose length lie in $[2^j, 2^{j+1})$; thus, the entire set of links is partitioned into disjoint length classes $L_0, L_1, \ldots, L_{\lfloor \log(d_{\max}) \rfloor}$, where $d_{\max}$ is the maximum link length.[8] For each non-empty length class j, the plane is divided into square grid cells of side $\mu 2^j$ each, where

$$\mu = 4\left(\frac{8\beta(\gamma-1)}{(\gamma-2)}\right)^{1/\gamma}. \qquad (24)$$

Interference "Constraint". For a cell A^j in L_j, let $\Delta(A^j)$ be the set of links in L_j whose receivers lie inside A^j. Now, consider a time slot t, and let $X_{e,k}$ be 1 iff e is active on channel k in t. In the proof of Theorem 5.2 of [14], it was shown that in such a setting, an optimal algorithm can only schedule a constant number (q) of links from any cell simultaneously. Thus, we have

$$\sum_{e \in \Delta(A^j)} X_{e,k} \quad \leq \quad q \qquad \forall\ j, A^j, \qquad (25)$$

$$\text{where } q \quad = \quad \frac{(2\sqrt{2}\mu + 2)^\gamma}{\beta}. \qquad (26)$$

Averaging Equation 25 over all time slots, we get:

$$\sum_{e \in \Delta(A^j)} \alpha(e,k) \leq q, \quad \forall j, A^j. \qquad (27)$$

LP Formulation, and Scaling. We formulate the LP again using Equations 13 to 21, but use Equation 27 instead of Equation 20. As before, we first solve the LP, and then scale down the resulting link utilizations by a factor of $(q+2)$. Let $\{\overline{\alpha}(e,k)\}$ be the scaled-down link utilizations. It is easy to see that $\overline{\alpha}$ values satisfy the following.

$$\forall\ e, j, A^j \quad \eta(\overline{\alpha}, e) + \sum_{e' \in \Delta(A^j)} \overline{\alpha}(e',k) \leq 1 \qquad (28)$$

Near-Optimal Link Schedule. Using techniques of [14], we can design a near-optimal link schedule for our JRCAS problem as follows.

- Pick a large enough integer W such that $W\overline{\alpha}(e,k)$ is a positive integer for each e and k.
- For each non-empty length class L_j, partition the plane into square grid cells of side $\mu 2^j$ each. Now, color the cells using 4 colors such that adjacent cells have different colors. Let L_{jh} be the set of links in L_j whose receiver lie in a h-colored cell.
- For each length class j and color h, we use a link schedule S_{jh} of length W and place links from L_{jh} into S_{jh} as follows. For each (e,k), such that $e \in L_{jh}$, we place the link e with channel k in the first $\overline{\alpha}(e,k)W$ time slots of S_{jh}, such that the interface constraint is not violated and no two links (e,k) and (e',k) are placed in the same time slot if $e, e' \in \Delta(A^j)$. From Theorem 5.1 of [14], such a placement ensures an interference-free link schedule. The feasibility of the above placement algorithm is shown in the below theorem.
- Now, concatenate the link schedules S_{jh} to get the full link schedule solution S.

Theorem 3: The above algorithm returns an $4(q+2)g(L)$-approximate solution for the JRCAS problem with physical interference and dynamic channel assignment. For the case of static channel assignment, the above gives a $4qg(L)K/I_{\min}$-approximation algorithm. Here, q is as defined in Equation 26 and $g(L)$ is the number of non-empty length classes.

6. For simplicity, we assume β to be a constant, i.e., the full link capacity can be used as long as the given physical interference constraint is satisfied.

7. Barring references to two specific results of [14]'s work, the following discussion is self-contained.

8. For ease of presentation, we assume that the minimum distance between any pair of nodes is at least 1. To meet such an assumption, link lengths are normalized and N is appropriately scaled.

Proof: First, we show the feasibility of the above described placement algorithm. For any pair (e,k), where $e \in \Delta(A^j)$, the maximum number of time slots wherein (e,k) can *not* be placed due to interference "constraint" is $W\sum_{e'\in\Delta(A^j)\setminus\{e\}}\overline{\alpha}(e',k)$. Now, by Equation 28 and similar arguments as in Theorem 2, the above placement algorithm is feasible.

The total length of S is $4g(L)W$ time slots, and hence, the link utilization of each pair (e,k) in S is $\overline{\alpha}(e,k)/4g(L) = \widehat{\alpha}(e,k)/(4(q+2)g(L))$ where $\widehat{\alpha}(e,k)$ is the link utilization of the LP solution. Thus, S is a $((q+2)4g(L))$-approximate solution. The claim for static channel assignment follows from similar arguments as before. ∎

Removing the $N = 0$ Assumption. We now show how to incorporate background noise N into the above technique. We categorize links based on their lengths d with respect to the quantity $(P/N\beta)^{1/\gamma}$, where P is the uniform transmission power.

1) Links with length greater than $(P/N\beta)^{1/\gamma}$: According to the SINR equation, such links are infeasible and can be safely ignored.
2) Links with length equal to $(P/N\beta)^{1/\gamma}$: Such links tolerate zero interference and hence, *must* be placed in a separate time slot of their own (even by the optimal algorithm).
3) Links with a length d such that
$$1 \leq d < 2^{\lceil \log(P/N\beta)^{1/\gamma}\rceil - 1}.$$
Such links lie in a length-class j, where $j < \lceil \log(P/N\beta)^{1/\gamma}\rceil - 1$ and thus, $1 - (N\beta 2^{(j+1)\gamma}/P) > 0$.[9] For each such length-class j, we redefine μ as:
$$\mu'_j = 4\left(\frac{8\beta(\gamma-1)}{(1-(N\beta 2^{(j+1)\gamma}/P))(\gamma-2)}\right)^{1/\gamma},$$
and use the same scheduling strategy as before with square grid cells of side $2^j\mu'_j$ each.
4) The remaining links have a length d such that
$$2^{\lceil \log(P/N\beta)^{1/\gamma}\rceil - 1} \leq d < (P/N\beta)^{1/\gamma},$$
and thus, are in the r^{th} length-class where $r = \lceil \log(P/N\beta)^{1/\gamma}\rceil - 1$. For such links, we use μ as:
$$\mu'_r = 2^{1+\epsilon}\left(\frac{8\beta(\gamma-1)}{(1-(N\beta d_{\max}^{\gamma}/P))(\gamma-2)}\right)^{1/\gamma},$$
where $\epsilon = \log_2 d_{\max} - \lfloor \log_2 d_{\max}\rfloor$ and $d_{\max}$ is the maximum length of a link less than $(P/N\beta)^{1/\gamma}$.

For the last two cases above, we can easily extend the proofs of Theorem 5.1 and 5.2 of [14]. Now, using similar arguments as before in this subsection, we get the following result.

Theorem 4: The above modified algorithm returns an $4(q'+2)g(L)$-approximate solution for the JRCAS problem with physical interference and dynamic channel assignment with non-zero noise. Here, $q' = \frac{(2\sqrt{2}\mu'+2)^\gamma}{\beta}$, where $\mu' = \max_{j\leq r}\mu'_j$ with $r = \lceil \log(P/N\beta)^{1/\gamma}\rceil - 1$ and μ'_j as defined above. For the case of static channel assignment, the above gives a $4q'g(L)K/I_{\min}$-approximation algorithm. ∎

The main shortcoming of the above result is that the approximation ratio depends on the P and N values. In the next subsection, we design an approximation scheme that does not suffer from this shortcoming. For the above length-class based scheme of this subsection, one way to remove the dependence of the approximation ratio on P and N is as follows. *First*, observe[10] that for each length-class $j \leq r-2$, $\mu'_j \leq 4\left(16\beta(\gamma-1)/(\gamma-2)\right)^{1/\gamma}$. Second, we restrict the optimal solution from using links in the length-classes r and $r-1$, by allowing the optimal solution to use slightly less transmission power (as in [9]). Then, the approximation-ratio of the above approach can be bounded by $4(q''+2)g(L)$, where q'' is defined in terms of the aforementioned bound on μ'_j.

4.2 Approximation Algorithm Based on Weights

In this subsection, we present an approximation algorithm for JRCAS with physical interference model, whose approximation ratio is independent of the transmission power and noise values. Our second approximation algorithm is based on the concept of weights which lets us represent the physical interference constraint as a linear equation. For a pair of *distinct* links (u,v) and (r,s), let
$$w_{(u,v)}^{(r,s)} = \frac{\beta P_v(r)}{P_v(u)}.$$
A similar concept of weights has been used in [16] to develop network throughput bounds and in [20] to develop scheduling heuristics without any performance bounds. Now, it is easy to see from the SINR equation that transmission along a link e is successful in presence of a set E' of other links if and only if
$$\sum_{e'\in E'} w_e^{e'} \leq 1 - (N\beta/P_e),$$

9. Since $j < \lceil \log(P/N\beta)^{1/\gamma}\rceil - 1$ and $j+1 \leq \lceil \log(P/N\beta)^{1/\gamma}\rceil - 1$, then, $2^{j+1} < (PN/\beta)^{1/\gamma}$.

10. For any length class j less than $r-1$, note that $2^{j+1} \leq (1/2)(P/N\beta)^{1/\gamma}$.

where $P_{e=(u,v)} = P_v(u)$ is the received power at v of the signal transmitted by node u

Let $\mathcal{C}$ be an upper bound on $\sum_{e' \in E'} w_e^{e'}$ for any E' and $e \in E$. That is, let $\mathcal{C}$ be such that

$$\sum_{e' \in E'} w_e^{e'} \leq \mathcal{C} \quad \forall\ e \in E, E' \subseteq E \setminus \{e\}. \tag{29}$$

We will later bound $\mathcal{C}$'s value under certain assumptions.

Physical Interference Constraint using Weights. Based on the above definition of $w_e^{e'}$ and $\mathcal{C}$, we can represent the physical interference constraint as follows. Consider a time slot t, and let $X_{e,k}$ be 1 if e is active on channel k in the time slot t and 0 otherwise. Now, for each e and k, the following holds.

$$X_{e,k} + \frac{1}{\mathcal{C}} \sum_{e' \in E \setminus \{e\}} w_e^{e'} X_{e',k} \leq 1 + \frac{1 - (N\beta/P_e)}{\mathcal{C}}.$$

To see the above, consider two cases: (i) When $X_{e,k}$ is 1, $\sum_{e' \in E \setminus \{e\}} w_e^{e'} X_{e',k} \leq 1 - (N\beta/P_e)$, and (ii) When $X_{e,k}$ is 0, use Equation 29. Now, averaging the above equation over all time slots, we get:

$$\alpha(e,k) + \frac{1}{\mathcal{C}} \sum_{e' \in E \setminus \{e\}} w_e^{e'} \alpha(e',k) \leq \left(1 + \frac{1 - (N\beta/P_e)}{\mathcal{C}}\right), \quad \forall\ e, k. \tag{30}$$

We use the above equation as the interference constraint in the LP formulation.

LP Formulation, and Near-Optimal Link Schedule. The LP formulation for the JRCAS problem with physical interference and multiple channels is the same as that for pairwise interference (i.e., Equations 13 to 21) except that we replace Equation 20 by Equation 30. As before, we first solve the LP optimally. Then, we scale down the LP solution's link utilizations by a factor of $(\mathcal{C}+3)$. Let the scaled-down link utilizations be $\{\overline{\alpha}(e,k)\}$. It is easy to see that the scaled-down link utilizations $\overline{\alpha}$ satisfy the following for all e, k.

$$\eta(\overline{\alpha}, e) + \overline{\alpha}(e,k) + \sum_{e' \in E \setminus \{e\}} w_e^{e'} \overline{\alpha}(e',k) \leq 1 \tag{31}$$

Now, as before, consider a link schedule S of appropriately chosen W time slots, and greedily place $\overline{\alpha}(e,k)W$ instances of (e,k) in S without causing any physical interference with or interface constraint violations with previously placed link instances.

Theorem 5: The above algorithm returns a $(\mathcal{C}+3)$-approximate solution for the JRCAS problem with physical interference and dynamic channel assignment. For the case of static channel assignment, there is a $((\mathcal{C}+1)K/I_{\min})$-approximation algorithm. Here, $\mathcal{C}$ is as defined in Equation 29.

Proof: Note that in a link schedule of W time slots, the number of time slots wherein a particular pair (e,k) can *not* be placed due to physical interference with previously placed links is at most $W \sum_{e' \in E \setminus \{e\}} w_e^{e'} \overline{\alpha}(e',k)$. Rest of the proof for dynamic channel assignment is same as that for Theorem 2. For static channel assignment, transform a $(\mathcal{C}+1)$-approximate single-channel solution as done for the pairwise interference case. ∎

Bounding $\mathcal{C}$. We now bound the value of $\mathcal{C}$, as defined by Equation 29. Let the minimum distance between any pair of nodes be $d_{\min}$. Then, the density ρ of nodes in the network is bounded by

$$\rho \leq 4/(\pi d_{\min}^2),$$

since disks of radii $d_{\min}/2$ placed at each node do not intersect. Now, assuming uniform transmission power P and log-distance path loss mode, we can bound the total signal strength at a node u due to all other nodes by integrating over the signal strength due to nodes in an annular disk of width dx at a distance of x from u as:

$$\int_{d_{\min}}^{\infty} \frac{P}{x^\gamma}(2\pi\rho x)\, dx = \frac{2P\pi\rho}{(\gamma-2)d_{\min}^{\gamma-2}} \leq \frac{8P}{(\gamma-2)d_{\min}^{\gamma}}$$

Note that the lower limit of the above integration is $d_{\min}$, since there are no nodes within a distance of $d_{\min}$ from u. Based on the above, the value of $\mathcal{C}$ can be bounded by:

$$\mathcal{C} \leq \frac{8\beta}{(\gamma-2)} \left(\frac{d_{\max}}{d_{\min}}\right)^\gamma,$$

where $d_{\max}$ is the maximum length of a link.

Comparing the Approximation Bounds. We note that the approximation bounds for both the above approximation algorithms for physical interference model can be quite large, depending on the exact parameter values. In fact, for non-zero background noise, the approximation bound of the length-based scheme of previous subsection can be arbitrarily bad due to its dependence on transmission power and noise values. With zero background noise, the value of q (Equation 26) for typical values of β (=10dB) and γ (=3) is around 25000 (independent of network topology). In contrast, the value of $\mathcal{C}$ depends much on network density and $d_{\max}/d_{\min}$ ratio. For the above typical values of β and γ, $\mathcal{C}$ is $80(d_{\max}/d_{\min})^3$, which may be much less than $4qg(L)$ for low values of $d_{\max}/d_{\min}$ but can also be much larger. We note that the proven approximation bounds are *provable worst-case* performance guarantees with respect to the *intractable optimal* solution. In fact, since both the algorithms do greedy scheduling at the lowest-level, they must certainly perform better than the best-known (naive) approach (interference-oblivious routing, followed by greedy channel-assignment and scheduling).

In our simulations (Section 6), we observed that weight-based approximation algorithm outperformed the length-class based approximation algorithm by a noticeable margin for a dense network of 100 nodes for varying number of channels and interfaces.

4.3 Physical Interference TDMA Link Scheduling Problem

We now use our techniques to design approximation algorithms for the TDMA link scheduling problem. Given a network graph with weighted links, the TDMA link scheduling problem is to design a link schedule S with minimum number of time slots such that S has w_e instances of each link e, where $w_e \geq 0$ is the given (integer) weight of e. To solve the above problem, we start with solving the following LP. Below, $W' = 1/W$, where W denotes the length of the desired link schedule.

$$\forall\ e,k, \qquad \alpha(e,k) \geq 0$$

$$\forall\ e, \qquad \sum_k \alpha(e,k) = w_e W' \tag{32}$$

$$\forall\ u \in V, \qquad \sum_{e \in N(u)} \sum_k \alpha(e,k) \leq I(u)$$

$$\forall e,k, \quad \alpha(e,k) + \frac{1}{\mathcal{C}} \sum_{e' \in E \setminus \{e\}} w_e^{e'} \alpha(e',k) \leq \left(1 + \frac{1-(N\beta/P_e)}{\mathcal{C}}\right)$$

$$\text{Maximize} \qquad W'$$

The second equation ensures that w_e instances of link e appear in the desired link schedule.

<u>$(\mathcal{C}+3)$-approximate Solution.</u> Let $\widehat{W}$ and $\{\widehat{\alpha}\}$ be the values of $W(=1/W')$ and α obtained from the LP solution. We now greedily place $\widehat{\alpha}(e,k)\widehat{W}$ copies of each pair (e,k) into a link schedule of length $\lceil \widehat{W}(\mathcal{C}+3) \rceil$ to derive a solution to the given TDMA link scheduling problem. The feasibility and approximation of the algorithm follows from the below theorem.

Theorem 6: The above algorithm returns a $(\mathcal{C}+3)$-approximate solution to the TDMA link scheduling problem with physical interference and dynamic channel assignment. For static channel assignment, a $(\mathcal{C}+1)$-approximation algorithm can be designed.

Proof: Note that $\sum_k \widehat{\alpha}(e,k)\widehat{W} = w_e$ by Equation 32. Thus, the above algorithm actually places w_e copies of each link in the derived solution. Moreover, the link utilizations in the designed link schedule are at most $1/(\mathcal{C}+3)$ fraction of that of the LP solution, and thus, they satisfy Equation 31. Thus, using similar arguments as before, the above placement algorithm is feasible, and the designed schedule S is $(\mathcal{C}+3)$-approximate. ∎

<u>$4(q+2)g(L)$-approximate Solution.</u> Similarly, if we use Equation 27 for the physical interference constraint, we will get a $4(q+2)g(L)$-approximate solution using techniques described before.

Theorem 7: Algorithm 1 of [14] can be generalized to deliver a $4(q+2)g(L)$-approximate solution for the TDMA link scheduling problem with physical interference model in multi-channel multi-radio networks with arbitrary link weights. ∎

5 JRCAS Problem with Single-Path Routing

In this section, we design approximation algorithms for the JRCAS problem with single-path routing, wherein traffic for each source-destination pair is restricted to a single path. We use randomized rounding technique by Raghavan and Thompson [29] and Chernoff's bounds [12] to solve the above problem. We assume uniform link capacity to obtain a closed-form expression for the approximation bound.

5.1 Single-Path Routing with Pairwise Interference

In this subsection, we assume pairwise interference. We consider physical interference in the next subsection.

Randomized Approximation Algorithm. Our algorithm consists of the following steps.

1) First, we solve the LP given by Equations 13 to 21. Let $\widehat{F}_i$ be the LP solution's data rate value of the i^{th} flow, i.e., the multi-path flow for (s_i, d_i) source-destination pair.
2) Using path-striping [29], we divide the i^{th} flow (of data rate $\widehat{F}_i$) into a sum/combination of single-path flows, each of value $\widehat{F}_i x_{ij}$ where $\sum_j x_{ij} = 1$. Here, x_{ij} is the fraction of the i^{th} flow that flows into the j^{th} single-path.
3) Next, we randomly round-off the fractional x_{ij} values to 0 or 1 as follows. For each i, exactly one x_{ij} is set to one (with a probability of x_{ij} each) and the rest are set to zero. The total flow of data rate $\widehat{F}_i$ is now routed unsplit through the single-path represented by x_{ij} that was set to 1. Note that the interface and

interference constraints may be violated in this step, but this is rectified by the following step.

4) Then, we scale down the link utilizations resulting from the above rounding-off process by a factor of $(c+2)\beta_{\max}$, where $\beta_{\max}$ (computed later) is a probabilistic upper-bound on the "inflation" (due to the randomized rounding) of the LHS (left hand side) expressions of interface and interference constraints (Equations 19 and 20).
5) Finally, we construct a link schedule for the scaled-down link utilizations using the greedy placement approach as in Section 3. We show below that the constructed link schedule is $(c+2)\beta_{\max}$-approximate with high probability.

Approximation Proof. We now prove the approximation ratio of the above algorithm. As suggested before, we first try to bound the inflation of the LHS expressions of interference and interface constraints due to the rounding-off process. To do so, we need to express the LHS expressions as a weighted summation of random variables, so that we can use generalization of well-known Chernoff's bounds to bound the summation. We start with some notations.

Notations. We use the following notations.

- $\Pr[.]$ to denote probability of an event, and $\mathbf{E}[.]$ to denote the expected value of a random variable.
- $\mathring{\alpha}$ to denote the link utilizations *after* the rounding-off process.
- X_{ij} to denote the binary *random variables* corresponding to the x_{ij} values, i.e., $\Pr[X_{ij}=1] = x_{ij}$ and $\Pr[X_{ij}=0] = 1-x_{ij}$.
- $\delta_{ij}(e,k)$ to denote the binary function, where $\delta_{ij}(e,k)$ is 1 iff the j^{th} single-path flow of i^{th} source-destination pair uses the link e with channel k, and is 0 otherwise.
- κ to denote the *uniform* link capacity.

Defining $\sum \mathring{\alpha}(e',k)$ as Weighted Sum of X_{ij}'s. We express $\sum_{e' \in C(e)} \mathring{\alpha}(e',k)$ as a weighted sum of X_{ij}'s as follows. Recall that, after the rounding-off process, for each i, the total flow of data rate $\widehat{F}_i$ is routed unsplit through the j^{th} single-path represented by X_{ij} that was set to 1. Thus, the link utilization $\mathring{\alpha}(e',k)$ after the rounding-off process can be represented by:

$$\mathring{\alpha}(e',k) = 1/\kappa \sum_i \left(\widehat{F}_i \sum_j \delta_{ij}(e',k) X_{ij} \right),$$

since $\delta_{ij}(e',k)$ (as defined above) determines whether the j^{th} single-path of i^{th} flow uses the link e' with channel k. Thus,

$$\begin{aligned} \sum_{e' \in C(e)} \mathring{\alpha}(e',k) &= \sum_{e' \in C(e)} \sum_{ij} (\widehat{F}_i/\kappa) \delta_{ij}(e',k) X_{ij} \\ &= \sum_{ij} \left(\sum_{e' \in C(e)} (\widehat{F}_i/\kappa) \delta_{ij}(e',k) \right) X_{ij} \\ &= \sum_{ij} a_{ij}(e,k) X_{ij}, \end{aligned} \tag{33}$$

where

$$a_{ij}(e,k) = (\widehat{F}_i/\kappa) \sum_{e' \in\ C(e)} \delta_{ij}(e',k).$$

We will bound the above summation using generalized Chernoff's bound. First, we make the following two observations.

1) Bounds on $a_{ij}(e,k)$.

$$0 \le a_{ij}(e,k) \le \widehat{F}_i D/\kappa, \tag{34}$$

where D is the maximum number of links in $C(e)$ used by a single-path.

2) $\mathbf{E}[\sum_{e' \in C(e)} \mathring{\alpha}(e',k)] \le c$, since

$$\begin{aligned} \mathbf{E}[\sum_{e' \in C(e)} \mathring{\alpha}(e',k)] &= \sum_{e' \in C(e)} \sum_{ij} (\widehat{F}_i/\kappa) \delta_{ij}(e',k) x_{ij} \\ &= \sum_{e' \in C(e)} \widehat{\alpha}(e',k) \\ &\le c \quad \text{(By Eqn. 20)} \end{aligned} \tag{35}$$

Above, $\widehat{\alpha}$ are the link utilization values of the LP solution (before the rounding-off process).

Generalized Chernoff's Bounds. We now state a slight generalization of the Chernoff's bounds for relative error (see Theorem 1 of [12]).

Theorem 8: Let X_{ij} be the above defined binary random variables, a_{ij} be non-negative real numbers, $X = \sum_{ij} a_{ij} X_{ij}$, and $\mu = \mathbf{E}[X]$. Then, the following bound holds for any $\delta > 0$,

$$\Pr[X > (1+\delta)\mu] < \left(\frac{e^{\delta}}{(1+\delta)^{(1+\delta)}} \right)^{\mu/a_{\max}},$$

where $a_{\max} = \max_{ij} a_{ij}$.

Proof: (Brief Sketch) The above follows from Exercise 4.14 of [26], and the following two facts:

1) It can be shown that $\mathbf{E}[\prod_{ij} e^{t a_{ij} X_{ij}}] \le \prod_{ij} \mathbf{E}[e^{t a_{ij} X_{ij}}]$. This follows from independence of X_{ij} and $X_{i'j'}$ for any $i \neq i'$, and the fact that for any particular i, only one X_{ij} is 1 and the rest are zero.

2) $\Pr(X \geq (1+\delta)\mu) = \Pr(X/a_{max} \geq (1+\delta)\mu/a_{max})$, which allows application of Exercise 4.14 of [26] result since $a_{ij}/a_{max} \in [0,1]$ for all i, j.

■

Bounding Inflation of $\sum \overset{\circ}{\alpha}(e', k)$ Expressions. The below lemma follows from Equations 33 to 35, and application of Theorem 8 to $\sum_{ij} a_{ij}(e,k)X_{ij}$.

Lemma 1: For any link e, channel k, and $\beta_1 \geq 2e-1$,

$$\Pr[\sum_{e' \in C(e)} \overset{\circ}{\alpha}(e', k) \geq (1+\beta_1)c] < 2^{-c\kappa\beta_1/D\widehat{F}_{\max}},$$

where $\widehat{F}_{\max} = \max_i \widehat{F}_i$ □

Similarly, if we define $b_{ij}(u)$ (for each u, i, j) as

$$b_{ij}(u) = \sum_k \sum_{e \in N(u)} (\widehat{F}_i/\kappa)\delta_{ij}(e,k)$$

and apply Theorem 8 to $\sum_{i,j} b_{ij}(u)X_{ij}$, we can show the following. Note that $\sum_j b_{ij}(u)X_{ij} \leq 2\widehat{F}_i/\kappa$.

Lemma 2: Let $\overset{\circ}{\alpha}(e) = \sum_k \overset{\circ}{\alpha}(e,k)$. $\forall\ u \in V, \beta_2 \geq 2e-1$,

$$\Pr[\sum_{e \in N(u)} \overset{\circ}{\alpha}(e) \geq (1+\beta_2)I(u)] < 2^{-I(u)\kappa\beta_2/2\widehat{F}_{\max}},$$

where $\widehat{F}_{\max} = \max_i \widehat{F}_i$ □

Approximation Result. Consider the link utilizations

$$\overline{\alpha}(e,k) = \frac{\overset{\circ}{\alpha}(e,k)}{(c+2)\beta_{\max}}, \quad \text{where}$$

$$\beta_{\max} = \max((1+\beta_1), (1+\beta_2), 2e-1), \text{ with}$$

$$\beta_1 = \frac{D\widehat{F}_{\max}\log(\frac{Q}{\epsilon})}{c\kappa}, \quad \beta_2 = \frac{2\widehat{F}_{\max}\log(\frac{Q}{\epsilon})}{I_{\max}\kappa}.$$

Above, $Q = K|E| + |V|$ is the total number of interface and interference equations, $I_{\max}$ is the maximum number of interfaces at a node, and ϵ is such that $0 \leq \epsilon \leq 1$. From the above two lemmas and union of probabilities, it is easy to see that the scaled-down link utilizations $\overline{\alpha}$ satisfy the sufficiency condition (Equation 23) with a probability of at least $(1-\epsilon)$. Also, note that $Q = O(Kn^2)$ where n is the network size, and $\widehat{F}_i \leq \kappa I_{\max}$. Thus, $c\beta_{\max} = O(DI_{\max}\log(n))$. Based on above, we have the following theorem.

Theorem 9: The above randomized algorithm delivers a $(c+2)\beta_{\max} = O(DI_{\max}\log(n))$-approximate solution with a probability of at least $(1-\epsilon)$ for the JRCAS problem with single-path routing and pairwise interference. Here, $\beta_{\max}, n, I_{\max}$ are as defined above, and D is as defined for Equation 34. ■

Above techniques extend to the generalizations (diversity, constraints, and objectives) outlined in Section 3.

5.2 Single-Path Routing with Physical Interference

For the single-path JRCAS problem with physical interference, we essentially follow the same approach as in the previous subsection, except for the following changes.

Length-Class Based Approach. For the approach based on length-classes, we make the following changes with respect to the previous subsection. Below, we assume zero background noise.

- In the LP, replace Equation 20 by Equation 27.
- Replace c by q in the expression for β_1.
- Define $\overline{\alpha}(e,k) = \overset{\circ}{\alpha}(e,k)/((q+2)\beta_{\max})$.
- Design the link schedule for the link utilizations $\overline{\alpha}(e,k)$ as described in Section 4.

Using similar arguments as before, we get the following.

Theorem 10: The above randomized algorithm delivers a $4(q+2)g(L)\beta_{\max}$-approximate solution with a probability of at least $(1-\epsilon)$ for the JRCAS problem with single-path routing and physical interference. ■

Non-zero background noise N can also be incorporated in the above result, as discussed in Section 4.1.

Weight-Based Approach. To show a similar approximation result for the randomized scheme based on weights and $\mathcal{C}$, we make the following changes with respect to the previous subsection.

- In the LP, replace Equation 20 by Equation 30.
- Define $\delta_{ij}(e, e', k)$ as follows.

$$\delta_{ij}(e,e',k) = \begin{cases} \mathcal{C}, & \text{if } (e,k) \in \mathcal{P}_{ij} \text{ and } e = e' \\ w_e^{e'}, & \text{if } (e,k) \in \mathcal{P}_{ij} \text{ and } e \neq e' \\ 0, & \text{otherwise} \end{cases}$$

  where $\mathcal{P}_{ij}$ is the *set* of links used by the j^{th} single-path of the i^{th} source-destination flow.
- $a_{ij}(e,k)$ is redefined accordingly.
- Replace c by $(\mathcal{C}+1)$ in the expression for β_1.
- Define $\overline{\alpha}(e,k) = \overset{\circ}{\alpha}(e,k)/((\mathcal{C}+3)\beta_{\max})$.
- Design the link schedule for the link utilizations $\overline{\alpha}(e,k)$ as described in Section 4.

Now, based on above definition of δ and a_{ij}, one can show that:

- For the new $a_{ij}(e,k)$, it is to see that

$$0 \leq a_{ij}(e,k) \leq 2\widehat{F}_i\mathcal{C}/\kappa \tag{36}$$

- $\mathbf{E}[\sum_{ij} a_{ij}(e,k)X_{ij}] \leq \mathcal{C}+1$, since

$$\begin{aligned}\mathbf{E}[\sum_{ij} a_{ij}(e,k)X_{ij}] &= \mathcal{C}\widehat{\alpha}(e,k) + \sum_{e'\in E\setminus\{e\}} w_e^{e'}\widehat{\alpha}(e',k)\\ &= \mathcal{C}\left(\widehat{\alpha}(e,k) + \frac{1}{\mathcal{C}}\sum_{e'\in E\setminus\{e\}} w_e^{e'}\widehat{\alpha}(e',k)\right)\\ &\leq \mathcal{C}+1 \quad \text{(By Eqn. 30)} \end{aligned} \tag{37}$$

  Above, $\widehat{\alpha}$ are the link utilization values of the LP solution (before the rounding-off process).

Finally, we use similar arguments as in Section 5.1, to get the following theorem.

Theorem 11: The above randomized algorithm delivers a $(\mathcal{C}+3)\beta_{\max}$-approximate solution with a probability of at least $(1-\epsilon)$ for the JRCAS problem with single-path routing and physical interference. ∎

6 Simulations Results

Our simulations have two objectives. First, for the JRCAS problem with physical interference (for both multi-path and single-path routing), we compare the performance of *weight-based* (based on Equation 30) and *length-class based* (based on Equation 27) algorithms. Second, for pairwise interference and single-path routing, we compare our randomized algorithm with a *Naive* approach (shortest-path routing followed by greedy assignment and scheduling).

Network Setup. Our simulations are conducted on a network of 100 nodes placed randomly in a region of 100×100 units. Each node has a transmission radius of 20 units, and two nodes are connected if they are within each other's transmission radius. Capacity of each link is one unit. We randomly select a set of 35 source-destination pairs and assign a traffic demand of 5 units to each pair. For pairwise interference, we use the secondary interference model.

Simulations. We consider three settings: pairwise interference with single-path routing (Figure 2), and physical interference with multi-path (Figure 3) and single-path routing (Figure 4). In each of the settings, we vary number of channels (with 10 interfaces/node) or number of interfaces per nodes (with 15 channels). We make the following observations.

- Increase in number of channels results in almost a proportional increase in the total data rate for all approaches and settings, except for the Naive algorithm in pairwise interference model.
- An almost similar trend is observed for increase in number of interfaces. However, as expected, increasing number of interfaces beyond a certain point does not increase the total data rate.
- For pairwise interference with single-path routing, our randomized algorithm outperforms the Naive approach especially for large number of channels where interference-awareness becomes more important.
- For physical interference model, the weight-based approach (based on Equation 30) outperforms the length-class based approach (based on Equation 27) for both multi-path and single-path routing.
- For the given network setup, the degradation of performance from multi-path to single-path routing is minimal.

7 Conclusions

In this article, we considered the joint routing (multi-path and single-path), channel assignment, and scheduling problem to maximize throughput in a network. Our unique contributions are design of approximation algorithms for the above joint problem in the context of dynamic channel assignment, physical interference model, and single-path routing. Our results extend the insightful technique of [22] to above general contexts. In future directions, we plan to extend of our techniques to handle non-orthogonal channels. Moreover, for the case of physical interference, we would like to design algorithms with better approximation bounds, by possibly considering restricted (but realistic) physical interference models. Design of distributed approximation algorithms for the above joint optimization problem remains a challenging open question.

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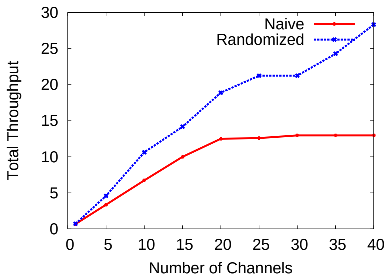
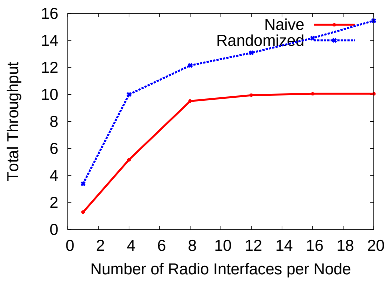
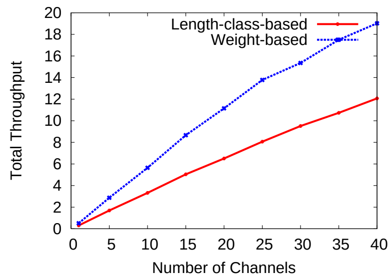


(a)

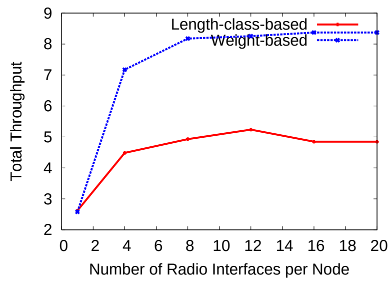


(b)

Fig. 2. Pairwise interference with single-path routing.

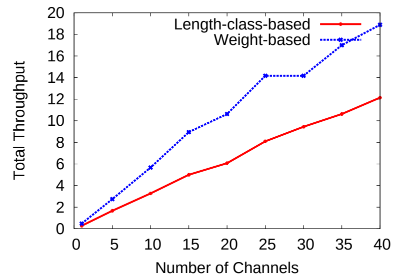


(a)

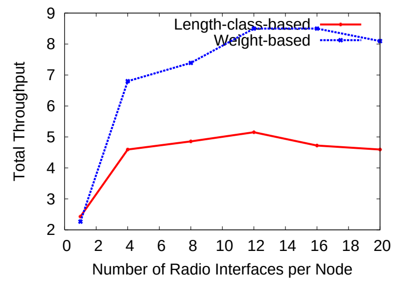


(b)

Fig. 3. Physical interference with multi-path routing.



(a)



(b)

Fig. 4. Physical interference with single-path routing.

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